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Propagation, localization, and control of light in Mathieu lattices

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Abstract

The main topic of this thesis is the examination of the propagation and control of light in Mathieu photonic lattices. The main directions of research are based on the formation of photonic lattices using single Mathieu beams or superposition of multiple Mathieu beams in a photorefractive crystal, then the propagation of light in photonic lattices thus formed, and the examination of the nonlinear propagation of single or elliptic Mathieu beams in a nonlinear photorefractive crystal.

The thesis is divided into seven chapters, and the content of the individual chapters is given in the following text.

The introductory chapter provides an overview of the results in the field of nonlinear photonics related to one of realizations of photonic crystals, photonic lattices. An overview of the known research and the achievements in the field of nonlinear optics and their contribution to other related fields are represented as well as future direction for research.

In the next chapter the photonic lattices are described, as well as achievements related to the investigation inside this thesis. The optical induction technique for the realization of photonic lattices is explained specifically by using propagation invariant light fields, i.e. nondiffracting beams.

In the third chapter nondiffracting beams are introduced with emphasis on the large group of Mathieu beams. Different families of Mathieu beams are shown: single even and odd Mathieu beams, elliptic and hyperbolic Mathieu beams. These beams will be used in this thesis to generate different photonic lattices in photorefractive media by optical induction.

The following chapter contains the description of the experimental method used for the research presented in this thesis to generate photonic lattices using Mathieu beams in a photorefractive strontium barium niobate crystal (SBN). Additionally, the light propagation in such generated Mathieu photonic lattices is described. This chapter contains the experimental explanation of photorefractive effect and refractive index modulation in photorefractive crystals as the main effect for optical induction of photonic lattices inside SBN crystal.

In the next chapter a numerical model for the examination of the propagation of the Mathieu light field, inside a photorefractive SBN crystal is described. Single Mathieu beams are used for experimental realization of photonic Mathieu lattices as well as interference of Mathieu beams, which produced numerous aperiodic lattices. A numerical model for the study of light propagation in photonic lattices created by nondiffracting Mathieu beams is also described.

In the next chapter, the results of the experimental and theoretical investigation are presented. This chapter is divided into three separate sections. In the first section, the nonlinear self-interaction of a single Mathieu beams in SBN crystal is investigated, numerically and experimentally. New effect, 1D and 2D nonlinear discrete diffraction are demonstrated as well as nonlinear Mathieu lattices. The nonlinear self-interaction of the elliptical Mathieu beams in the SBN crystal is examined and their utilization for the realization of dynamical structures which rotates in direction predicted with energy flow, i.e. Poynting vector. Such rotating dynamic structures are suited for the realization of chiral waveguides with the possibility of controlling the chirality and number of the waveguides. The second section of this chapter contains an examination of the propagation of an elliptical vortex beam in photonic lattices created by single Mathieu beams in SBN crystal. The conditions for the formation of stable vortex states such elliptical vortex necklaces, how the order or ellipticity of Mathieu beam, size and topological charge of the elliptical vortex beam influence the stability of vortex states. The third section an approach for the realization of two-dimensional aperiodic lattices using the interference of several Mathieu beams is presented. Therefore, the propagation of a narrow probe beam is examined
in such a formed lattice. It is investigated how the local environment will affect the diffraction of the probe beam in the lattice as well as the conditions for the formation of localized states in such lattices. The propagation of light in aperiodic lattices is compared with propagation in appropriate periodic lattices. The periodic square lattice with a period equal to the characteristic structured size of Mathieu beam is used.

The last chapter summarizes the results of the research and their potential application in other related fields. Due to the examination of nonlinear propagation of Mathieu beams in the photorefractive SBN crystal new effect, nonlinear discrete diffraction is revealed. Examination of elliptical Mathieu beams significantly contributed to the realization of chiral two-dimensional photonic lattices with adjustable properties via the optical induction technique facilitated by the elliptical Mathieu beams. The conditions during which Mathieu beams remain robust in the crystal are shown, as well as the various photonic lattices created by Mathieu beams, with waveguides located along straight and curved paths (circle, ellipse or hyperbola), nonlinear Mathieu lattices, chiral lattices, as same as different aperiodic lattices created via interference of Mathieu beams. In Mathieu lattices, the propagation of elliptic vortex is examined and various stable states such as elliptical vortex necklaces are found, as well as the conditions under which they remain stable. In aperiodic Mathieu lattices propagation of narrow Gaussian beam is examined the same as parameters for realization of strong localized states like spatial solitons.

**Key words:** Mathieu beams, nonlinear fotorefractive crystal, aperiodic photonic lattices, nonlinear photonic lattices, chiral photonic lattice, nonlinear morphing discrete diffraction, energy flow, vortices, vortex necklaces, solitons.

**Scientific field:** Physics

**Research area:** Nonlinear photonics
Rezime

Tema ove teze je ispitivanje prostiranja i kontrole svetlosti u Matjeovim rešetkama. Osnovni pravci istraživanja usmereni su ka formiranju fotonskih rešetki korišćenjem pojedinačnih Matjeovih zraka ili superpozicije više Matjeovih zraka u fotorefaktivnom kristalu, zatim ka prostiranju svetlosti u tako formiranim fotonskim rešetkama kao i ispitivanju nelinearnog prostiranja pojedinačnih ili eliptičnih Matjeovih zraka u nelinearnom fotorefaktivnom kristalu.

Rad je podeljen u sedam poglavlja, a sadržaj pojedinačnih poglavlja dat je u daljem tekstu.

U uvodnom poglavlju dat je pregled dosadašnjih rezultata u oblasti nelinearne fotonike povezanih sa fotonskim kristalima odnosno fotonskim rešetkama kao jedne od realizacija fotonskih kristala. Dat je pregled istraživanja, i osvrta na dostignuća u oblasti nelinearne optike i njihov doprinos u drugim srodnim oblastima kao i pravci za buduća istraživanja.

U sledećem poglavlju opisane su fotonske rešetke kao i neke od važnijih dostignuća u toj oblasti povezane sa istraživanjem u ovoj tezi. Objasnjen je tehnika optičke indukcije za formiranje fotonskih rešetki korišćenjem zraka nedifragujućih tokom propagacije tj. nedifragujućih zraka.

U trećem poglavlju opisani su nedifragujući zraci, a posebno jedna obimna grupa Matjeovih zraka. Prikazani su različite grupe Matjeovih zraka: pojedinačni parni i neparni Matjeovi zraki, Eliptični i Hiperbolici Matjeovi zraki, koje će se u ovoj tezi koristiti za izradu različitih fotonskih rešetki.

U sledećem poglavlju opisana je experimentalna metoda korišćena u toku istraživanja, čiji rezultati su prikazana u ovoj tezi, za formiranja fotonskih rešetki pomoću Matjeovih zraka u fotorefaktivnom kristalu stroncijum barijum niobatu (SBN). Takodje je opisana i metoda za ispitivanje prostiranja svetlosti u takom formiranim Matjeovim rešetkama. Ovo poglavlje sadrži eksperimentalno objašnjenje fotorefaktivnog efekta i modulacije indeksa prelamanja u fotorefaktivnom kristalu kao glavnih efekata pri upisivanju fotonske rešetke.

U narednom poglavlju opisan je numerički model kojim se opisuje prostiranje Matjeovog svetlosnog polja u fotorefaktivnom SBN kristalu, koji se koristi i za kreiranje Matjeovih rešetki. Matjeove rešetke kreirane su korišćenjem pojedinačnih Matjeovih zraka ili prethodne interferencije više Matjeovih zraka koji stvaraju brojne aperiodične rešetke. Potom je opisan numerički model za izučavanje linearnog i nelinearnog prostiranja svetlosti u fotonskim rešetkama kreiranim pomoću nedifragujućih Matjeovih zraka.

U narednom poglavlju prikazani su rezultati eksperimentalnog i teorijskog istraživanja. Ovo poglavlje je podeljeno na tri zasebne celine. U prvoj celini ispitivana je nelinearna samo-interakcija pojedinačnog Matjeovog zraka u SBN kristalu, numerički i eksperimentalno. Pokazan je novi efekat, 1D and 2D nelinearna diskretna difrakcija kao i nelinearne Mathjeove rešetke. Ispitana je nelinearna samo-interakcija eliptičnih Matjeovih zraka u SBN kristalu kao i njihova upotreba za realizaciju dinamičke strukture koje rotiraju u tokom prostiranja u pravcu protoka energije, koji je određen Pointingovim vektorom. Takve rotirajuće dinamičke strukture pogodne su za izgradnju talasovoda sa mogućnošću kontrolisanja zakrivljenosti i broja talasovoda. Druga celina ovog poglavlja sadrži ispitivanje prostiranja eliptičnog vorteksnog zraka u fotonskim rešetkama kreiranim pomoću pojedinačnih Mathjeovih zraka. Ispitivani su uslovi za formiranje stabilnih vorteksnih stanja kao što su vorteksne ogrlice, kako red ili eliptičnost Matjeovog zraka ili širana i toplološko naelktiranje eliptičnog vorteksa utiču na stabilnost vorteksnih stanja. U trećoj celini je prikazan pristup za generisanje novih dvodimenzionalnih aperiodičnih rešetki pomocu interferencije više Matjeovih zraka. U tako formiranim rešetkama ispitivano je prostiranje uskog probnog zraka. Ispitivano je kako lokalno okruženje...
utiče na difrakciju probnog zraka u rešetci kao i uslova za formiranje lokalizovanih stanja u takvim rešetkama. Prostiranje svetlosti u aperiodičnim rešetkama poredjeno je sa prostiranjem svetlosti u odgovarajućoj periodičnoj rešetki. Korišćena je periodična kvadratna rešetka sa periodom jednakim karakterističnoj veličini Matjeovog zraka korišćenog za izradu aperiodične rešetke.

U poslednjem poglavlju sumirani su rezultati istraživanja i njihova potencijalna primena u drugim srodnim oblastima. Prilikom ispitivanja nelinearnog prostiranja Matjeovih zraka u fotorefraktivnom SBN kristalu otkriven je novi efekat, nelinearna diskretna difrakcija. Ispitivanje eliptičnih Matjeovih zraka omogućio je značajan doprinos za realizaciju hiralnih dvodimenzionalnih fotonskih rešetki sa podesivim osobinama pomoću optički indukovane tehnike olakšane korišćenjem eliptičnih Matjeovih zraka. Pokazani su uslovi tokom kojih Matjeovi zrak ostaje robustan u kristalu kao i različite fotonske rešetke kreirane pomoću Matjeovih zraka sa talasovodima raspoređenim duž prave ili krive linije (krug, elipsa ili hiperbola), nelinearne Matjeove rešetke, hiralne Matjeove rešetke, kao i brojne aperiodične rešetke kreirane interferencijom Matjeovih zraka. U Matjevim rešetkama ispitivano je prostiranje eliptičnog vorteksa u kojima su pokazana stabilna vorteksna stanja kao što su eliptične vorteksne ogrlice kao i uslovi pod kojima ostaju stabilne. U aperiodičnim Matjeovim rešetkama ispitana je propagacija uskog Gausjanskog zraka i parametri za nastanak jako lokalizovanih stanja kao što su prostorni solitoni.

Ključne reči: Matjeovi zraki, nelinearni fotorefraktivni kristal, aperiodične fotonske rešetke, nelinearne fotonske rešetke, zakrivljene fotonske rešetke, neinearna diskretna difrakcija, protok energije, vorteksi, vorteksne ogrlice, solitoni.

Naučna oblast: Fizika

Uža naučna oblast: Nelinearna fotonika
Zahvalnica
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Chapter 1

Introduction

Research in the field of photonics involves the overlap of three basic scientific disciplines that are leading in today’s technological development: electronics, optics, and material physics. The results of such research are applicable in many fields, and during the years the scope and level of research and investment in this area are explosively increasing. The physics of semiconductors had the main role in information and communication technologies, but the advances are accomplished during the past decade by using a new class of materials - photonic crystals [1]. They are optical materials with a periodically modulated refractive index. According to the refractive index variations and period in space, there are one-dimensional (1D), and Bragg reflectors, dielectric mirrors, thin films, dielectric Fabry-Perot filters, etc.) are some of examples, two-dimensional (2D) e.g. commercially used holey fibers, and three-dimensional (3D) (Yablonovite, wood-pile, or opal geometry structures, etc.) photonic crystals. They have been used to control light propagation and emission. Because of their periodic nature, photonic crystals form allowed and forbidden photonic bands, the same as electronic bands in semiconductors. It is shown that light in photonic crystals propagates like an electron in semiconductors with periodic potential. Thus, photonic crystals can be considered analogs of semiconductor materials. Besides, photonic crystals are noticed in nature in different forms. But their artificial realization started from Lord Rayleigh, permit investigation of light propagation phenomena promising useful applications in other fields.

Photonic lattices are one of the realizations of photonic crystals, they represent optical waveguides with periodic changes in the refractive index of the medium, with low refractive index contrast. The studies about the photonic lattices are modeled by the paraxial equation formally identical to the Schrödinger equation, which describes crystal lattices of solid-state physics. In photonic lattices propagation coordinate $z$ is equivalent to the time coordinate in the Schrödinger equation. Periodical refractive index change, provides photonic band gaps in photonic lattices. They are recognized as electronic band gaps of crystal lattices of solid-state physics where the periodic potential of the atoms leads to forbidden regions in the transmission spectrum. The photonic band gaps show forbidden regions for electromagnetic wave propagate.

An important feature of photonic lattices is the ability to manipulate light propagation in the direction of periodicity. In photonic lattices, phenomena analogous to phenomena from solid-state physics such as Bloch oscillations, Zener tunneling, Anderson localization, etc. are found. To increase the control opportunities of electromagnetic wave nonlinear optical materials e.g. nonlinear photorefractive crystals are used. In such materials light-matter interaction is established, allowing light to influence the optical properties of materials and thus the light itself. Due to the nonlinearity and periodic change of the refractive index in the photonic lattices, a competition of linear and nonlinear phenomena occurs and self-trapping lights, the appearance of spatial, or lattices solitons, modulation instabilities, etc. are observed in photonic lattices.
The optical induction technique allows the realization of different photonic lattices inside a photosensitive material. Photonic lattices, characterized by refractive index modulation in a certain direction opposite to translationally invariance through the direction of propagation are frequently realized in different photorefractive crystals. Nondiffracting beams due to their propagation invariant transverse intensity distributions have been utilized in the realization of photonic lattices by optical induction technique. Their transverse spatial frequency components lie on a circle in the corresponding Fourier plane, to demonstrate their nondiffracting character [2]. They are exact solutions of the Helmholtz equation in different coordinate systems. Depending on the underlying real space coordinate system, mostly, four families of nondiffracting beams are used for optical induction of photonic lattices: in Cartesian coordinate system discrete nondiffracting beams (plane wave interference patterns i.e. discrete beams ), in cylindrical coordinates Bessel beams, in elliptic cylindrical coordinate system Mathieu beams, and in parabolic cylindrical coordinates system parabolic beams. Besides nondiffracting character, they possess other peculiar characteristics like robustness and self-healing, and they had been used in different fields such as atomic optics, optical tweezers, nonlinear optics, etc.

Until now, different researches about photonic lattices as well as control and manipulation of light in photonic lattices are done. In periodic photonic lattices, it is shown that control of light propagation is determined by bandgap properties [3]. The discrete diffraction behavior of the evanescently coupled waveguide arrays (1D periodic waveguide arrays) was examined first. In weakly guiding waveguides the light was readily confined at discrete sites, or light would be exchanged through waveguides via coupling during propagation. When only one waveguide is initially excited by light, diffraction is different from that occurring in continuous systems, the most of the light energy is carried out along two major lobes far from the center excited waveguide. This diffraction pattern today is known as discrete diffraction [4]. Also, discrete diffraction is observed in two-dimensional photonic lattices [5]. The optical solitons in waveguides arrays are explained as a balance between nonlinearity of waveguides and discrete diffraction effects arising from linear coupling among adjacent waveguides [6, 7]. Light is nonlinearly confined in a few waveguides and it can propagate diffractionless. Optical discrete solitons in two-dimensional nonlinear waveguide lattices were observed in photorefractive crystals [5, 8].

Research in the field of photonics has been extended from the periodic lattices and the propagation of light in them to the study of disordered photonic structures [9, 10] and deterministic aperiodic structures, which are at the transition between periodic and disordered structures [11]. Disorder in photonic systems creates weak or strong localization of light, known as Andersen localization and coherent backscattering [12, 13, 14]. Deterministic aperiodic structures, discovered 1984, are the structures with fifth-order symmetry, which was considered nonexistent in crystals, and a long-range order that was considered nonexistent in amorphous bodies [15]. When quasicrystals are found in solid-state physics, many new research fields are open in optics and photonics [16, 17, 18]. Quasicrystals are unique structures that can be viewed as extensions of the periodic crystal concept in which translational symmetry is replaced by long-range order. Sharp diffraction patterns confirm the existence of wave interference as a consequence of long-range order [19]. They do not have transitional symmetry, so it is not possible to define a unit cell. However, due to the long-range order, phenomena characteristic for periodic structures, such as Brag diffraction, are observed in quasicrystals. They can be formed by a substitution paving rule based on two or more building blocks. The most famous example of these structures is the Penrose lattice, and over time various other schemes have been developed to form new kinds of quasicrystals (Fibonacci, Thue-Morse, Rudin-Shapiro, etc.).
Light control is crucial for many scientific fields from classical nonlinear optics to plasma physics, Bose-Einstein condensate, solid-state physics, and recently in information technology. The leading trend is the search for optical analogs for electronically integrated circuits that enable routing, control, and processing of optical signals. Deterministic aperiodic photonic structures are appropriate for the control and manipulation of light and future utilization in optical devices. Control and manipulation of light in such specially designed structures is an active topic of research in optic and photonics.

The realization of various aperiodic structures and their further investigation are some of the main motivations for research in this thesis. The versatility of aperiodic structures is very important and provides considerable flexibility and richness in modeling the optical response. Propagation of light in such structures is still an exciting area for researchers and essential for the application of future devices. Nondiffracting Mathieu beams are outstanding candidates for the creation of new aperiodic photonic lattices. The interference of Mathieu beams in a different spatial disposition one to another provides an approach realization of aperiodic patterns and their optical induction in photosensitive media to induce aperiodic lattice. Moreover, further examination of light propagation in these aperiodic lattices in the linear and nonlinear regimes is one of the main topics in this thesis.

Straight waveguides were the first and simplest step for the examination of light propagation. Further, waveguides with bending in the longitudinal direction, periodically oscillating waveguides, or chiral waveguides have been realized and manipulation of light propagation was examined in some of them [20, 21, 22]. Two-dimensional twisted lattices still are a challenge in experimental realization and this is a very appealing area of research. They require compound experimental setup, longer experimental realization time, or expensive equipment. The realization of chiral structures using the optical induction technique and Mathieu beams is the motivation for research in this thesis. Elliptical Mathieu beams would be investigated in different nonlinear regimes to inspect if they can create stable or dynamic structures. The realization of 2D twisted dynamic structures inside nonlinear photorefractive crystal via elliptical Mathieu beams would be a substantial contribution to the simplify realization of chiral lattices by optical induction.
Chapter 2

Photonic lattices

Photonic crystals exist in nature, however, they can be realized artificial. Resulting in previous research their properties are a great replacement for semiconductors. Researchers around the world investigate the propagation of electromagnetic waves in photonic crystals as well as the potential application of photonic crystals in technological development. Photonic crystal fibers, i.e. holey fibers and photonic lattices are some of the realizations of photonic crystals. Holey fibers are applied in many areas like fiber-optic communications, fiber lasers, nonlinear devices, etc., therefore, already present in our everyday life.

Photonic lattices are artificial structures with periodical refractive index modulation, with low contrast of refractive index compared to photonic crystals. 1D photonic waveguides are periodic in one transversal direction and invariant through the longitudinal direction. 2D photonic lattices are characterized by periodical modulation in both transversal directions and invariant through the longitudinal direction, while 3D photonic lattices have periodically modulation in whole three dimensions. Propagation of light differs in photonic lattices then in homogeneous media, so they are powerful tools to manipulate light propagation. Many new phenomena, like discrete diffraction, lattice and gap solitons, harmonic generation, stimulated scattering etc. are allowed in them.

According to many examinations about photonic lattices, which possess Brillouin zones, allowed and forbidden bands, and so on, it is observed analogous between light propagation in photonic lattices and motion of electrons in semiconductors [23]. Electromagnetic wave propagates in photonic lattices the same as electron in crystal lattices of solid-state physics. Many experimental studies of photonic lattices are done to prove and other phenomena predicted by quantum mechanics like Bloch oscillations [24], Zener tunneling [25] and Anderson localization [13]. Investigation of light propagation in photonic lattices is explained by the paraxial equation which is the analog of the quantum-mechanical Schrödinger equation but time coordinate is replaced with propagation coordinate $z$ [26].

In the past, various periodic photonic structures have been exploited to control the propagation properties of electromagnetic waves. 1D arrays of evanescently coupled optical waveguides are created with an equal distance between waveguides, with all characteristics of a photonic crystal structure (Brillouin zones, band structure, etc.) [27]. In such systems (Fig. 2.1 (A1)), light couples between waveguides through tunneling, showing its diffraction characteristics. When low-intensity light is injected into one or a few neighboring waveguides, it couples to more and more further waveguides, broadening its spatial distribution. This is a new physical effect in comparison to diffraction in homogeneous media. In the linear regime, light diffracts from inner excited waveguide to outer waveguides. This characteristic diffraction pattern is known as discrete diffraction and the same effect is observed in both 1D and 2D periodic photonic lattices as shown in Fig. 2.1 [7, 27, 28].
Nonlinear optic, as an active research area includes fundamental studies of light-matter interactions to numerous optical applications. The high-intensity light of the probe beam produces nonlinear responses of light in photosensitive material. When high-intensity light excites into one or a few neighboring waveguides light diffraction is confined with the nonlinearity of media and spatial and lattice solitons were achieved in 1D and 2D periodic photonic lattices [3, 30, 31].

Propagation of light in periodic lattices results in many fascinating effects but using disorder lattices and photonic quasi-crystals, which are in between periodic and disorder structures, the new fields of research are opened. In the last decade’s examination of disorder photonic lattices acquire the attention. Since the first experimental observation of Anderson localization, new investigations about the Anderson localization in disorder systems were revealed like Anderson localization of light near boundaries of disorder lattices or dimensionality switching and with the continuous transformation of the lattice structure from one-dimensional to two-dimensional were done [9, 32, 33, 34, 35].

Examination of light propagation in photonic quasi-crystals, and deterministic aperiodic structures like Penrose quasicrystals, Fibonacci arrays, Thue-Morse, Rudin-Shapiro sequences, Vogel lattice etc. [16, 19, 36, 11], extensively increase. Localization of light is observed in some of them. The quasi-crystals with the increasing disorder as well as light propagation in them were examined. In such structures, light localization is observed, while enhancing wave transport was established. However, new approaches for the realization of different aperiodic lattices and light expansion in them is still an open question in optics. Such researches would provide fundamental explanations of light transport in aperiodic lattices.
2.1 Photonic lattices and the optical induction technique

There are several methods used for the experimental realization of photonic lattices, like waveguides detached in semiconductors [37], lithography, lattices optically induced in photosensitive media [8], laser-written arrays in silica [38] etc.

The optical induction technique is significant because allows an arbitrary structuring of photorefractive materials and fast and simple erase of such structures with white light. By this technique, it is possible to generate different photonic lattices: periodic, aperiodic, or disordered for fundamental investigations of wave propagation in such structures. The basic idea of the optical induction technique is to modulate the refractive index of nonlinear material by external illumination (photorefractive effect). By spatial light modulator, the linearly polarized light wave (lattice beam) is spatially modulated. Thus, refractive index modulation into the biased photosensitive crystal is induce and photonic lattices is generated into the crystal. Such photonic lattices are suited for study of additional probe beam propagation.

Some of the photorefractive materials, such as strontium barium niobate crystal SBN crystal, show a strong polarization anisotropy, where the strength of the nonlinearity is dependent on a light wave polarization (equivalently the corresponding electro-optic coefficient) [39]. Ordinary polarized light wave in SBN propagates in an effectively linear regime, does not show any self-action, but induces the required refractive index modulation i.e. photonic lattices. An extraordinary polarized light wave, in general, used as a probe beam, feels different nonlinear response of such crystal in the dependency of the beam power (nonlinearity strength). For low-intensity, probe beam feels low nonlinearity strength, while if the intensity of the probe beam is sufficiently increased, it modifies the refractive index and propagates nonlinearly through crystal. Photonic lattices, optically induced in SBN crystal, are used for studying linear and nonlinear propagation effects in dependency on the probe beam power.

In order to achieve desired 1D or 2D photonic structures using the optical induction technique, the transversally periodic intensity distribution, invariant along the propagation directions are used. Nondiffracting beams are convenient for the realization of photonic lattices by optical induction technique due to their transverse-invariant propagation. They can conveyance their intensity distribution to the complete length of the photorefractive crystals. Nondiffracting beams are used for optically induction of different periodic structures (strip, square, diamond, hexagonal, etc. patterns), a new class of photonic structures like Bessel, Mathieu, or Weber lattices, quasi-periodic Penrose lattice, or deterministic aperiodic structures like Vogel or Fibonacci lattices, etc. Also, three-dimensional periodical lattices and helical structures are produced by using nondiffracting beams and reconfigurable optical induction method [40, 41]. Mathieu beams are one of the nondiffracting beams with diversity intensity distribution. So far such beams are only in few realization of photonic lattices. In this thesis, Mathieu beams would be examined for optical induction of different photonic lattices.
Chapter 3

Nondiffracting beams

Nondiffracting beams are monochromatic optical fields with propagation-invariant transverse intensity distributions. The term nondiffracting beam was introduced by Durnin in 1987 for the propagation in vacuum [42]. They were examined as exact solutions of the homogeneous Helmholtz equation in different coordinate systems [42, 43, 44]. Nondiffracting beams were obtained in the system of the cylindrical coordinates under the limitation that their complex amplitude is separable as the product of the functions which depend on the transverse coordinates and propagation coordinate [42]. The transverse amplitude profile of such beams was mathematically described by the Bessel functions and they are named Bessel beams. Later, different kinds of nondiffracting beams were introduced [45, 46] and their properties and applications are examined.

Nondiffracting beams with finite energy are reviled bounded or by the homogeneously transmitting aperture of finite dimensions or by the Gaussian aperture, known as pseudo-nondiffracting beams. Numerically and experimentally investigations show that the nondiffracting and pseudo-nondiffracting beams own the sharp $\delta$-like angular spectrum, represented by a circle in the Fourier plane, which proves their propagation-invariant character [47]. Propagation-invariant characteristics of such beams make them useful in optical micromanipulation [48], nonlinear optics [8, 3], wireless communication, etc.

Beside completely eliminated diffraction of the such beams, additional properties, helpful for potential applications are detected. One highly valuable property is the robustness of the nondiffracting beams [49, 50]. It was shown that the nondiffracting beam shows the resistance of the against amplitude and phase distortions, also they are able to regenerate its intensity profile to the original form in the free propagation behind the nontransparent obstacle [51], verified by the simple experiment [47]. Nondiffracting beams also possess a self-reconstruction ability (The Talbot effect and self-imaging) [52]. Efficient methods of generating nondiffracting beams require the use of the computer-generated holograms [53], the axicon [54] or the programmable spatial light modulators [55].

Two-dimensional scalar time-independent Helmholtz equation:

$$\nabla^2 U(x, y) + k^2 U(x, y) = 0$$

(3.0.1)

is separable in 11 coordinate systems but admits separation into the transverse and longitudinal parts only in four coordinate systems: Cartesian, circular cylindrical, elliptical cylindrical, and parabolic cylindrical coordinates [2]. Each of these coordinate systems gives rise to a certain type of nondiffracting beam such as discrete (plane wave and superposition of plane waves), Bessel, Mathieu, and Parabolic beams, respectively.

Periodic intensity patterns are generated as a superposition of the plane waves. Plane waves whose transverse Fourier components are located on the same ring determined by their wave vectors,
create stationary intensity distribution along the propagation direction. One-dimensional periodic intensity distribution would be achieved if two plane waves have interfered, also two-dimensional periodic (square, hexagonal, etc.) intensity can be created by the superposition of multiple plane waves [40, 56, 8]. Moreover, quasiperiodic pattern e.g. Penrose lattice is generated with the superposition of five plane waves [19, 57, 58, 59].

By transforming and solving the Helmholtz EQ. (3.0.1) in circular cylindrical coordinates Bessel beams are obtained and their transverse intensity distributions are characterized by concentric rings [42]. Fig. 3.1 (A) depicts a zero-order Basel beam. In recent studies, Bessel beams were applied to induce periodic lattices, and even 2D Fibonacci lattices [60].

Mathieu beams are solutions of the Helmholtz equation in the elliptical coordinates system [45]. Their transverse intensity distribution consists of discrete spots along elliptic or hyperbolic paths. The numerically calculated transverse intensity distribution for the fourth-order Mathieu beam is shown in Fig. 3.1 (B).

Parabolic beams are the fourth family of nondiffracting beams. They are the solution of the Helmholtz equation in the parabolic cylindrical coordinate system [46]. In Fig. 3.1 (C) transverse intensity distribution of the Parabolic beam with the continuous parameter \( a = 0 \) is depicted.

Figure 3.1: Intensity distributions of (A) Bessel, (B) Mathieu and (C) Parabolic nondiffracting beams.

### 3.1 Mathieu functions

Plenty of scientific and engineering problems lead to differential equations of the Mathieu type. Solutions of these equations are known as Mathieu functions. First, they are analyzed by Mathieu in 1868 [61]. Later these functions were further investigated in numerous investigations like the motion of an electron in a one-dimensional potential [62], wave propagation in periodic structure [63], analysis of vibrating modes in the elliptical membrane [64], or used for the formulation of invariant optical fields [45].

Mathieu equations originate from the separation of Helmholtz equation in elliptical cylindrical coordinates. By transformation rectangular coordinates \((x, y)\) to elliptical cylindrical coordinates \((\xi, \eta)\) by coordinate transformation

\[
x + iy = f \cosh(\xi + i\eta)
\]

\((x = f \cosh(\xi) \cos(\eta), y = f \sinh(\xi) \sin(\eta))\) with corresponding Laplacian transformation EQ. (3.0.1)
is transformed to the **two-dimensional Helmholtz equation in elliptic coordinates**

\[
\left[ \partial^2_{\xi} + \partial^2_{\eta} + \frac{f^2 k_t^2}{2} \cos(2\xi) - \cos(2\eta) \right] U(\xi, \eta) = 0. \tag{3.1.3}
\]

The solutions of EQ. (3.1.3) are separable as the product of the functions which depend on the elliptical cylindrical coordinates \((\xi, \eta)\) as \(U(\xi, \eta) = R(\xi)A(\eta)\). The functions \(R(\xi)\) and \(A(\eta)\) must satisfy the equations

\[
\left[ d^2_\xi + (a - 2q \cosh(2\xi)) \right] R(\xi) = 0, \tag{3.1.4}
\]

\[
\left[ d^2_\eta + (a - 2q \cos(2\eta)) \right] A(\eta) = 0. \tag{3.1.5}
\]

where \(q\) is a dimensionless parameter related to the transverse propagation constant \(k_t\) as \(q = \frac{f^2 k_t^2}{4}\), where \(f\) is semi-focal distance and \(a\) is the separation constant arising from the separation of variables method.

In physics and engineering literature EQs. (3.1.4) and (3.1.5) are known as the **Radial Mathieu Equation**, and the **Angular Mathieu Equation**, respectively. Their solutions are the **Radial Mathieu Functions** and the **Angular Mathieu Functions**. This nomenclature originates from the similarity between elliptical \((\xi, \eta)\) and polar coordinates: the elliptic variable \(\eta\) has a domain \(0 \leq \eta < 2\pi\) and plays a similar role to a polar angle, whereas the variable \(\xi\), with domain \(0 \leq \xi < \infty\) behaves as a radial variable (Fig. 3.2).

In this thesis, Mathieu functions are used to create an enormous family of Mathieu nondiffracting beams.

![Figure 3.2: Elliptic coordinate system. Curves \(\xi = \) constant are confocal ellipses, curves \(\eta = \) constant are orthogonal hyperbolas; \(0 \leq \eta < 2\pi, 0 \leq \xi < \infty\). For limit \(f \to 0\) polar coordinates are review.](image-url)
3.2 Mathieu nondiffracting beams

Among diverse families of nondiffracting beams, Mathieu beams may be interpreted as a generalized beam class, which interpolate between Cartesian and spherical coordinates [45, 46, 65, 66]. Their transverse spatial intensity distributions can form paths on ellipses or hyperbola. Mathieu functions are used for mathematical observation of Mathieu beams.

**Single Mathieu beams** of order \( m \) are mathematically described by a product of Radial and Angular Mathieu functions of order \( m \), introduced in the previous section. Single Mathieu beams have two parity: **even** (e) and **odd** (o) represented as [67]

\[
M^{e}_m(\xi, \eta) = C_m(q) J^e_m(\xi; q) \, c^e_m(\eta; q), \tag{3.2.6}
\]

\[
M^{o}_m(\xi, \eta) = S_m(q) J^o_m(\xi; q) \, s^o_m(\eta; q). \tag{3.2.7}
\]

where \( C_m(q) \) and \( S_m(q) \) are weighting constants, depend on *parameter of ellipticity* \( q = \frac{f^2 k_t^2}{4} \) related with position \( f \) of the two foci and transverse wave number \( k_t = \frac{2\pi}{a} \), where \( a \) is **characteristic beams size**. \( J^e_m \) and \( J^o_m \) are even and odd Radial Mathieu functions of order \( m \), and ellipticity \( q \). \( c^e_m \) and \( s^o_m \) are even and odd Angular Mathieu functions of order \( m \), and ellipticity \( q \).

Order of even Mathieu beams starts from zero, in contrast, odd Mathieu beams stars from order one. In Figs. 3.3 and 3.4 there are shown even and odd Mathieu beams of different order \( m \), with the same parameter of ellipticity \( q = 25 \), and characteristic structure size \( a = 25\mu m \). As the order of Mathieu beams increases, intensity distributions become more complex, because they are separated in compound sites arranged over different paths straight or curved lines.

![Figure 3.3: Intensity and phase distribution of even Mathieu beams of different order \( m \), with same parameter of ellipticity \( q = 25 \), and characteristic structure size \( a = 25\mu m \).](image)
Mathieu beams rely on the elliptic cylindrical coordinate system, which is reflected in their transverse intensity and phase distributions. From phase distributions shown in Figs. 3.3 and 3.4, separate regions of different phases values on the ellipses and hyperbolas with joint foci are noticeable, therefore the elliptical character of Mathieu beams displayed.

Figure 3.4: Intensity and phase distribution of odd Mathieu beams of different order $m$, with same parameter of ellipticity $q$, and same characteristic structure size $a = 25\mu m$.

Figure 3.5: Intensity and phase distribution of even Mathieu beam of order $m = 8$, with characteristic structure size $a = 25\mu m$ and different parameter of ellipticity $q$. 
Parameter of ellipticity changes the shape of Mathieu beam. The shape of even Mathieu beam of order $m = 8$ and structure size $a = 25\mu m$ is examined according to increasing different values of ellipticity $q = [0, 25, 125, 325]$ as depicted in Fig. 3.5 while characteristic structure size $a$ influence on the size of Mathieu beams.

Next, the influence of beams size $a$ is examined. Even Mathieu beam of order $m = 4$ and ellipticity $q = 25$ for different structure sizes $a = [6.25, 12.5, 25, 50]\mu m$ are depicted in Fig. 3.6. While the characteristic size of Mathieu beams increases, the distance between the sites increases too.

![Figure 3.6: Intensity and phase distribution of even Mathieu beam of order $m = 4$, with parameter of ellipticity $q = 25$ and different characteristic structure size $a$.](image)

A complex superposition of even and odd Mathieu beams of the same order $m$ represents elliptic Mathieu (ELM) beams which intensity and phase distribution are shown in Fig. 3.7. For a monochromatic, scalar elliptic Mathieu beam of order $m$, the light field is given by

\[
ELM(\xi, \eta) = M_m^e + iM_m^o.
\]

(3.2.8)

The transversal intensity distributions of elliptic Mathieu beams are distinguished by a series concentric ellipses, while the phase distributions are continuously modulated along ellipses.
Figure 3.7: Intensity and phase distribution of *elliptic Mathieu beams* of different order $m$, with same parameter of ellipticity $q$, and same characteristic structure size $a=25\mu m$.

A complex superposition of even Mathieu beams of order $m$ and odd Mathieu beams of the order $(m + 1)$ represents *hyperbolic Mathieu beams* which intensity and phase distribution are shown in Fig. 3.8.

\[ H_{ym}(\xi, \eta) = M^e_m + iM^{o}_{m+1} \]  \hspace{1cm} (3.2.9)

The transversal intensity distributions of Hyperbolic Mathieu beams are distinguished by a series of hyperbolas, while the phase distributions are continuously modulated along hyperbolas.

Figure 3.8: Intensity and phase distribution of *Hyperbolic Mathieu beams* of different order $m$, with same parameter of ellipticity $q$, and same characteristic structure size $a=25\mu m$. 

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Scalar even and odd Mathieu beams exhibit only real-valued field distributions, therefore their transverse Poynting vector vanishes. In contrast, the elliptic Mathieu beams are complex spatial modulated beams, owing to the occurrence as a complex superposition of even and odd Mathieu beam of the same order, showing outstanding continuously modulated spatial phase distributions, i.e., orbital angular momentum [66, 68, 69]. Thus, for these beams a transverse energy flow is present, described by their Poynting vector.
Chapter 4

Experimental methods for photonic lattices generation and light propagation in photonic lattices

In this thesis, nondiffracting Mathieu beams are used as light fields for optical induction of refractive index modulations in 15mm or 20mm long photorefractive SBN crystal. Moreover, linear and nonlinear propagation of single Mathieu beams and elliptical Mathieu beams is investigated in such crystal.

This chapter presents mechanisms of photorefractive effect and refractive index modulation in photorefractive SBN crystal. Two experimental realizations are illustrated, one for experimental realization of photonic Mathieu lattices by optical induction, and the other for experimental investigation linear and nonlinear light propagation of the probe beam (plane wave, Gaussian or elliptical vortex) in such photonic lattices optically induced in a nonlinear photorefractive SBN crystal.

4.1 Photorefractive effect

Photorefractive optics is a field of nonlinear optics important in scientific research as well as in technological applications. This field of physics studies the phenomena associated with the propagation of laser beams in nonlinear photorefractive materials. Photorefractive materials have been known for 40 years. Mostly, these are translucent and anisotropic ferroelectric crystals pure or with certain impurities whose energy states are in between the valence and the conduction band. Impurities are select to be acceptors or donors for the crystal. When the photorefractive crystal is illuminated with the light of the appropriate wavelength some of the electrons or holes from donors or acceptors are photoionized and they elevate from the valence to the conduction zone (Fig. 4.1). Free electrons and holes are generated in illuminated areas. Diffusion or drift can affect the movement of charges in the conduction zone.
In diffusion, mechanism charges are recombined with empty donors or traps. Static space-charge field $E_{sc}$ is build up by such recombination process. This space charge field creates the refractive index modulation via the linear electro-optic effect (the Pockels effect). The strength of the refractive index modulation depends on the illumination intensity and the intrinsic parameters of the crystal.

The second mechanism for index modulation is the drift. Opposite to the diffusion, the drift occurs only if an external electric field is applied to the crystal. Excited electrons and holes are accelerated by the external field and move until the force created with the external field is compensated by the internal field, which is created by the resulting inhomogeneous charge distribution. Electrons are more drifted than holes. Drift distance of electrons depends on the external field. In general, a combination of both effects exists with an external field, but in this thesis, drift effect is much stronger than diffusion, hence the effects of diffusion mechanism would be neglected in some examinations.

The photorefractive effect has several distinctive properties: dependence of doped element, it is the highly sensitive effect (observed at low light intensities), slow effect (depend on the light intensity, doped element mobility, and external field intensity). In some photorefractive materials, the refractive index modulation is highly persistent in the dark. Also, the refractive index modulation is erasable by homogenous illumination like LED light or high temperature, without crystals damaged.

The photorefractive effect is found in several classes of electro-optic materials such as barium titanate (BaTiO3), lithium niobate (LiNbO3), zinc telluride (ZnTe), potassium niobate (KNbO3), strontium barium niobate (SBN), organic photorefractive materials, certain photopolymers, and some multiple quantum well structures. Such crystals are used for frequency filtering in the field of mobile communications, as electro-optic modulators, in nonlinear photonics, etc. In photorefractive crystals, holograms [70] or photonic lattices [39, 8] are formed.
4.1.1 The linear electro-optic effect

The photorefractive effect depends on the linear electro-optic effect i.e. Pockels effect as mentioned in the previous section. This effect describes the change of the refractive index of a material induced by the presence of a static electric field $E_{sc}$ [71, 72, 73]. This is a second-order nonlinear effect related to second-order nonlinear susceptibility. The refractive index change is described by the impermeability tensor $\hat{\eta}$ i.e. the inverse of the dielectric permittivity tensor $\hat{\varepsilon}$ as

$$\Delta \hat{\eta}_{i,j} = \Delta \hat{\varepsilon}^{-1} = \Delta \left( \frac{1}{n_0^2} \right)_{i,j} = \sum_{k=1}^{3} r_{i,j,k} E_k^{sc}$$

where, $r_{i,j,k}$ is the linear electro-optic tensor, $E_k^{sc}$ is the applied electric field and $k, i, j = 1, 2, 3$ (or $x, y, z$). The dielectric permittivity tensor $\hat{\varepsilon}$ is a symmetric tensor of rank 2 (matrix 3 x 3)

$$\hat{\varepsilon} = \varepsilon_0 n_0^2 = \varepsilon_0 \begin{bmatrix} (n_0^e)^2 & 0 & 0 \\ 0 & (n_0^o)^2 & 0 \\ 0 & 0 & (n_0^e)^2 \end{bmatrix},$$

$$\hat{\eta} n_0^2 = 1$$

where $n_0^o, n_0^e$ are unperturbed refractive indices of ordinary and extraordinary polarization, respectively. The linear electro-optic tensor $r_{i,j,k}$ is third-rank tensor, which 27 components can be reduced to 18 independent, because $\hat{\varepsilon}$ and $\hat{\eta}$ are symmetric so indices $i$ and $j$ may be interchanged $r_{i,j,k} = r_{j,i,k}$. In this manner, new notation for linear electro-optic tensor is introduced

$$r_{1k} = r_{11k},$$
$$r_{2k} = r_{22k},$$
$$r_{3k} = r_{33k},$$
$$r_{4k} = r_{23k} = r_{32k},$$
$$r_{5k} = r_{13k} = r_{31k},$$
$$r_{6k} = r_{12k} = r_{21k}.$$  

Some of these components are equal to zero or identical in many of the photorefractive crystals in dependency on the crystal point group symmetry.

4.2 Properties of photorefractive SBN crystal

In optics different photonic lattices have been optically induced in photorefractive media and use for further investigation of light propagation. In photorefractive SBN crystal, due to his properties, refractive index changes are controlled locally by an external electric field, thereby allowing the realization of adaptive waveguides and complex photonic structures by light.

SBN crystal ($Sb_x Ba_{1-x} Nb_2O_6, 0.25 \leq x \leq 0.75$) communally pure and doped with Ce, Cr, Co, Fe is an excellent optical and photorefractive material frequently used in electro-optics, acousto-optics or photorefractive nonlinear optics. Pure SBN crystals are used in optical information storage and investigations of relaxor phase transitions. Megumi et al. were the first who discovered a noticeable...
improvement of the photorefractive properties when doping SBN with cerium [70]. SBN crystal was
doped by adding different amounts of CeO$_2$ to the melt. The nowadays growing technique (Modified
Stepanov technique [74]) provides outstanding optical quality crystals with a definite cross-section
and linear dimensions up to 100 mm. This unique crystal growing technique allows fabrication of
high-quality SBN crystal with particularly large electro-optic, thermo-optic, pyro-electric and piezo-
electric coefficients, and excellent optical quality. The possibility of inducing reversible refraction
index modulation by inhomogeneous illumination (photorefractive effect) is the most important char-
acteristic of SBN crystal. Such crystal possesses huge flexibility, provided since inscribed structures
can easily be erased with homogeneous white light illumination.

SBN is a birefringent, uniaxial and anisotropic material with crystallographic symmetry 4mm. For SBN
crystal dominant effect that leads to a refractive index modulation is linear electro-optic
effect, where refractive index modulation is related to the linear electro-optic coefficients. Due to the
point group symmetry of 4mm the only non-vanishing electro-optic coefficients for SBN are $r_{13}$, $r_{33}$,
and $r_{42}$, where especially $r_{13}, r_{42} \ll r_{33}$:

\[
\begin{bmatrix}
0 & 0 & r_{13} \\
0 & 0 & r_{13} \\
0 & 0 & r_{33} \\
r_{42} & 0 & 0 \\
r_{42} & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]  \quad (4.2.10)

Due to this new relation for the change of impermeability tensor EQ. (4.1.1) in SBN crystal is define
as

\[
\Delta \hat{n} = \Delta \left( \frac{1}{n_0^2} \right) = \begin{bmatrix}
r_{13} E_z^{sc} & 0 & r_{42} E_x^{sc} \\
0 & r_{13} E_z^{sc} & r_{42} E_y^{sc} \\
r_{42} E_y^{sc} & r_{42} E_y^{sc} & r_{33} E_z^{sc}
\end{bmatrix}
\]  \quad (4.2.11)

Biased on EQ. (4.2.11), the refractive index change is defined as $\Delta n^2 = -n^2(\Delta \hat{n})n^2$ or

\[
\Delta n^2 = -\begin{bmatrix}
(n_0^e)^4 r_{13} E_z^{sc} & 0 & (n_0^e)^2 (n_0^e)^2 r_{42} E_x^{sc} \\
0 & (n_0^e)^4 r_{13} E_z^{sc} & (n_0^e)^2 (n_0^e)^2 r_{42} E_y^{sc} \\
(n_0^e)^2 (n_0^e)^2 r_{42} E_x^{sc} & (n_0^e)^2 (n_0^e)^2 r_{42} E_y^{sc} & (n_0^e)^4 r_{33} E_z^{sc}
\end{bmatrix}
\]  \quad (4.2.12)

The refractive index can be separate on part without light $n_0^2$ i.e. refractive index of SBN
crystal and part with refractive index change $\Delta n$ induces by light. Expression for $\Delta n$ is obtained by
the approximation $n^2 = n_0^2 + \Delta n^2 = (n_0 + \Delta n)^2 = n_0^2 + 2n_0^2 \Delta n + O[\Delta n]^2$. Since $\Delta n^2 \ll n$
quadratic term $\Delta n^2$, can be neglected and refractive index change is obtained

\[
\Delta n \approx -\frac{1}{2} \begin{bmatrix}
(n_0^e)^3 r_{13} E_z^{sc} & 0 & (n_0^e)(n_0^e)^2 r_{42} E_x^{sc} \\
0 & (n_0^e)^3 r_{13} E_z^{sc} & (n_0^e)(n_0^e)^2 r_{42} E_y^{sc} \\
(n_0^e)^2 (n_0^e) r_{42} E_x^{sc} & (n_0^e)^2 (n_0^e) r_{42} E_y^{sc} & (n_0^e)^3 r_{33} E_z^{sc}
\end{bmatrix}
\]  \quad (4.2.13)
For SBN crystal, the optical $c$-axis coincides with the $x$-axis (Fig. (4.2)), same the direction the electric field $E_{sc} = E_{sc} \cdot e_x$. SBN crystal shows strong polarization anisotropy so the index change for a light that is ordinary (normal to $c$-axis) polarized is much smaller than for extraordinary polarized light. According to EQ. (4.2.13), the following expressions for the refractive index change induced by of ordinary and extraordinary light are

$$\Delta n^o \approx -\frac{1}{2} n_0^o r_{13} E_{sc}, \quad (4.2.14)$$

$$\Delta n^e \approx -\frac{1}{2} n_0^e r_{33} E_{sc}. \quad (4.2.15)$$

Figure 4.2: Geometry of SBN crystal and axis orientation.

SBN ($Sb_{0.6}Ba_{0.4}Nb_2O_6$) crystal doped with 0.002 wt. % CeO$_2$, with typical geometrical dimensions of $5 \times 5 \times 15$mm$^3$ or $5 \times 5 \times 20$mm$^3$ is used in this thesis for experimental realizations as well as in numerical simulations. Such crystal is characterized with unperturbed refractive indices $n_0^o = 2.325$ and $n_0^e = 2.358$ and corresponding the electro-optic coefficients $r_{13} = 47.1$ pm/V and $r_{33} = 237.0$ pm/V for ordinary and extraordinary polarization, respectively [75].
4.3 Experimental realization of photonic lattices

Photonic lattices can be created by modulating the refractive index of the medium, which includes direct laser writing, optical lithography, or drilling techniques. A very practical method is the optical induction technique in photorefractive material that produces permanent, reversible photonic structures represented by the intensity profile of a nondiffracting light field. It has an advantage over other techniques because it is simply possible to write and erase structures without permanently damaging the crystal. Photorefractive SBN crystal is a suitable material to optical induce photonic lattice. The basic conception of the optical induction technique in SBN crystal is to modulate the refractive index by external illumination (photorefractive effect). According to recalculated modulation illumination, different refractive index modulations are acquired. This technique was used for generation of periodic lattices [8], aperiodic lattices [60, 76], random lattices [9] or dielectric structures by artificially designed refractive index modulation, i.e. helical twisted photonic lattices [41].

Figure 4.3 presents the experimental setup for the optical induction of photonic lattices in SBN crystal. As a light source is used the frequency-doubled Nd: YVO$_4$ (neodymium-doped yttrium orthovanadate) laser which gives continuously light with a wavelength $\lambda = 532$nm. The laser beam is expanded and collimated to illuminate spatial light modulator (SLM) as a plane wave (Fig. 4.3). SLM (Holoeye Pluto VIS) is a phase-only modulator that has full HD $1920 \times 1080$ px$^2$ displays, with a pixel size of $8 \times 8 \ \mu$m$^2$ and dimensions of $11.25 \times 8.64$ mm$^2$, created from Liquid Crystal on Silicon. By SLM the reflected light field is modulated in both amplitude and phase [77]. Different paraxial scalar light fields are used for refractive index modulation by addressing a precalculated hologram to the SLM containing the information of the complex light field encoded with an additional blazed grating. The telescope L1 - L2 scales down the SLM size by a factor of 10 and by applying an appropriate Fourier filter inside this telescope the tailored complex light field is realized [77, 78]. The SBN crystal shown in Fig. 4.2 is installed in the experimental setup with its optical c-axis perpendicular to the direction of propagation, declared to be the z-axis. The SBN crystal is placed in the beam path, therefore the modulation is envisioned at the front face of the crystal.

![Nd:YVO$_4$ laser](lambda=532nm)

**Figure 4.3:** Experimental setup for realization of photonic lattices in SBN crystal: SLM - spatial light modulator, BS - beam splitter, L - lens, FF - Fourier filter, MO - macroscopic objective.

The crystal is externally biased with an electric dc field of $E_{ext}$ aligned along the optical $c = x$-axis, perpendicular to the direction of propagation ($z$-axis) via electrodes as depicted in Fig. 4.2. The ordinary polarized light linearly polarized in the y-direction, address the electro-optic coefficient
$r_{13}$ on SBN crystal. Such structure beams are used for experimental examination of linear propagation or optical induction of photonic lattices via Mathieu beams. The extraordinary polarized light linearly polarized in the x-direction, addressing the electro-optic coefficient $r_{33}$ of SBN crystal controls the nonlinear response of SBN crystal. Due to this the extraordinary polarized beam is used for experimental examination of nonlinear propagation of Mathieu beams in SBN crystal. Typically, the strength of the electric field is $E_{ext} = 0.8 - 2$ kV/cm. By this redistribution of charge is fast and distinguished refractive index modulations in the order of $\Delta n_{max} = 10^{-4}$ are achieved.

Behind the SBN crystal, an imaging system with a microscope objective (MO) and the camera is placed. The imaging system is mobile in the z-direction. The back face of the photorefractive crystal is imaged by a microscope objective onto the camera. Because the refractive index of the material is spatially modulated the light distribution inside the SBN crystal can not be imaged from experimental realization. Numerical simulations are used to predict the experimental realization. When experimental and numerical results have a good agreement, numerical simulations are used to present light distributions inside the crystal.

### 4.4 Experimental realization of light propagation in photonic lattices

For the investigation of light propagation in photonic lattices, the optical induction technique has been extended in the photorefractive SBN crystal by using ordinary polarized writing beam for lattice induction and extraordinary for probe beam [39], because SBN crystal feels different nonlinearity strength according to light polarization depending on the electro-optic coefficient $r_{ij}$. Ordinary polarized light with appropriate electro-optic coefficient $r_{13}$ propagates in almost linear regime, while extraordinary polarized light with higher electro-optic coefficient $r_{33} \gg r_{13}$ feels stronger nonlinearity.
The experimental setup for the investigation of light propagation in photonic lattices is shown in Fig. 4.4. The new setup is more complex then the previous one. The same laser source is used. The continuous laser beam is split via beam splitter into two separate beams: the ordinary polarized writing beam and the extraordinary polarized probing beam. The writing beam is used for optical induction of refractive index modulations in a photorefractive SBN crystal in the same way as in the previous section. Both beams are spatially tailored in intensity and phase by two phase-only SLMs. For this purpose, pre-encoded holograms are addressed to the SLMs Holoeye Pluto and Heo and their diffraction patterns are bandpass filtered in Fourier space (FF₁ & FF₂). Both SLMs have full HD resolution displays, with a pixel size of $8 \times 8\mu m^2$ and dimensions of $11.25 \times 8.64 mm^2$. By telescopes (L₁ - L₂ and L₃ - L₄) the SLMs size is scaled down and both spatially tailored light beams and superimposed with beam splitter $BS₃$ to send modulated light to the front face of the SBN crystal.

SBN crystal is externally biased with an electric field aligned along the optical $c = x$-axis, perpendicular to the direction of propagation, the $z$-axis parallel to the long axis of the crystal presented in Fig. (4.2). Probing the artificial photonic structure is done with the extraordinary polarized probe beam that addresses the stronger electro-optic coefficient $r_{33}$ responsible for the nonlinear response. Strength of nonlinearity is determined by applied external field $E_{ext}$ and laser power $P$. Typically, the strength of the electric field is $E_{ext} = 0.8 - 2$ kV/cm and distinguished refractive index modulations is in the order of $\Delta n_{max} = 10^{-4}$.

An imaging system made of a microscope objective and camera detects transverse intensity distributions at the back of the crystal. Intensity distributions of the probe beam or writing beam could be recorded. While the intensity distribution of the probe beam at the back face of the crystal is recording the writing beam and external field are turned off and vice versa.

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Figure 4.4: Experimental setup for light propagation in optically induced photonic lattices in SBN crystal: SLM - spatial light modulator, BS - beam splitter, L - lens, FF - Fourier filter, MO - macroscopic objective.
Chapter 5

Numerical tools for photonic lattices
generation and light propagation in photonic lattices

In this chapter, the numerical model for the examination of Mathieu light beams propagation and creation of Mathieu lattices in photorefractive SBN crystal is presented. Afterwards, the numerical model for investigation of light propagation in such created photonic lattices by using Mathieu beams is introduced.

Such numerical methods are used to simulate earlier presented experiments. But in experiments exist some limits like the length of the crystal, inability to record the crystal volume, hence numerical simulations are a good opportunity to present requirements that are not feasible in experimental realization.

5.1 Basic equation of light propagation in nonlinear photorefractive media

In the purpose of examination in this thesis, investigation of light propagation in nonlinear photorefractive media starts from the Maxwell’s equations in regions containing no free charges or currents, \( \rho = 0 \) and \( J = 0 \):

\[
\frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H},
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E},
\]

\[
\nabla \cdot \mathbf{B} = 0,
\]

\[
\nabla \cdot \mathbf{D} = 0.
\]

where \( \mathbf{D} \) is the electric displacement, \( \mathbf{B} \) is the magnetic induction, \( \mathbf{E} \) is the electric field, and \( \mathbf{H} \) is the magnetic field. They are related by following equations:

\[
\mathbf{D} = \varepsilon_0 \cdot \mathbf{E} + \mathbf{P},
\]

\[
\mathbf{B} = \mu_0 \cdot \mathbf{H}
\]
where $\epsilon_0$, and $\mu_0$ denotes the permittivity and permeability of free space, and $P$ the induced polarization of the material. By mathematical calculation the wave equation of electric field is obtained

$$\nabla \times \nabla \times \mathbf{E} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\frac{1}{\varepsilon_0 c^2} \frac{\partial^2 \mathbf{P}}{\partial t^2}, \tag{5.1.7}$$

where $c=\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ is the speed of light in vacuum.

For nonlinear photorefractive media, polarization of light is dependent on the electric field as $\mathbf{P} = \varepsilon_0 \chi_{\text{eff}} \mathbf{E}$, where $\chi_{\text{eff}}(I = |E|^2)$ is effective intensity dependent susceptibility. According to this the electric displacement and effective refractive index are define as $\mathbf{D} = \varepsilon_0 (1 + \chi_{\text{eff}}) \mathbf{E}$, $n(I) = \sqrt{1 + \chi_{\text{eff}}}$. The wave equation (5.1.7) can be rewrite according to the relation $\nabla \times \nabla \times \mathbf{E} = \nabla (\nabla \mathbf{E}) - \nabla^2 \mathbf{E}$ as

$$\nabla (\nabla \mathbf{E}) - \nabla^2 \mathbf{E} = -\frac{n^2(I)}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}. \tag{5.1.8}$$

Term $\nabla^2 \mathbf{E}$ is small for most cases of interest and can be neglected, thus, the Helmholtz equation is obtained

$$-\nabla^2 \mathbf{E} + \frac{n^2(I)}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0. \tag{5.1.9}$$

Propagation of light in photorefractive media induce refractive index change, according to that refractive index $n^2(I) = n_0^2 + \Delta n^2(I)$ is divided into two parts: the part without light $n_0^2$ (refractive index of the crystal) and part with refractive index change $\Delta n$ induces by light. Through this thesis, the propagation of the light field is in the $z$-direction, examined with linearly polarized light, thus, electric field is specified via

$$\mathbf{E}(\mathbf{r}, t) = U(\mathbf{r})e^{(k_z z - \omega t)} \mathbf{e}_x \tag{5.1.10}$$

with longitudinal wavevector $k_z = n_0 k_0$, $k_0 = \frac{\omega}{c}$, and $\mathbf{r} = \{x, y, z\}$.

In the standard paraxial approximation, the wave envelope slowly varying in the $z$-direction in contrast with the fast oscillating part of the wave in the $z$-direction, $|k_z \partial_z U| \gg |\partial^2_z U|$. A significance of paraxial approximation is to permit that the double partial $z$ derivative of the envelope can be neglected $|\partial^2_z U| \approx 0$. EQ. (5.1.10) is substituted in EQ. (5.1.9) with respect of paraxial approximation, resulting in two-dimensional nonlinear paraxial Schrödinger equation (NPE)

$$i \frac{\partial U(\mathbf{r})}{\partial z} + \frac{1}{2k_z} \nabla^2_\perp U(\mathbf{r}) + \frac{k_z}{2n_0^2} \Delta n^2(I) U(\mathbf{r}) = 0, \tag{5.1.11}$$

with $\nabla^2_\perp = \left( \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} \right)$. The EQ. (5.1.11) describes paraxial light wave propagation $(k_z^2 + k_x^2 \ll k_z^2)$ and includes nonlinear response of media. Light propagates in the $z$-direction, linearly polarized, parallel to the optical $c$-axis of the crystal, with slowly varying wave envelope $U(\mathbf{r})$ in the $z$-direction on a scale much more longer then the wavelength $\lambda = 2\pi/k_z$.

By NPE (EQ. (5.1.11)) linear and nonlinear light propagation phenomena in photorefractive media can be described. In photorefractive material, main effect for refractive index modulation is the linear electro-optic effect in combination with charge transport mechanisms and requires an external electric field. The presence of a dc field leads to the change in the dielectric permittivity, thus light induced refractive index change $\Delta n$ in dependence of the electro-optic coefficient $r_{ijk}$ and the electric field $E$ (EQ. (4.2.13)). The band transport model describes the dynamics of the charge...
The total electric field \( E \) and charge field \( I \) light intensity \( E \)

\[ \text{sum of the external light field and the space-charge field is expressed in terms of its electrostatic potential} \]

In this thesis diffusion effect is neglected \( D = 0 \). Therefore, the potential equation is simplified

\[ \Delta \phi_{sc} + \nabla \ln(1 + I) \nabla \phi_{sc} = E_{ext} \frac{\partial}{\partial x} \ln(1 + I) - D \left[ \Delta \ln(1 + I) + (\nabla \ln(1 + I))^2 \right]. \quad (5.1.12) \]

In 2D case the potential equation has no analytical solution, therefore it has to be solved numerically. Afterward, the resulting potential can be used to calculate the total electric field and the refractive index modulation dependent on initial intensity distribution.

In some cases of this thesis, the diffusion effect is taken into account particularly when the strong nonlinearities are applied along the direction of the optical \( c \)-axis. Due to the diffusion shift, asymmetry in intensity distribution is noticed along the optical \( c \)-axis of the crystal. But in some examination in this thesis diffusion effect is neglected \( D = 0 \). Therefore, the potential equation is simplified

\[ \Delta \phi_{sc} + \nabla \ln(1 + I) \nabla \phi_{sc} = E_{ext} \frac{\partial}{\partial x} \ln(1 + I), \quad (5.1.13) \]

and the space-charge field is express in terms of its electrostatic potential \( E_{sc} = \partial_x \phi_{sc} \). Due to this, the total electric field \( E(I) = E_{ext} + E_{sc}(I) \) that builds up inside the photorefractive crystal is the sum of the external light field \( E_{ext} \) and internal space charge field \( E_{sc} \) which depends on the initial light intensity \( I = |U|^2 \). The potential equation carries all information about the photorefractive nonlinearity i.e. the dependency \( E_{sc}(I) \). Photorefractive nonlinearity is saturable due to bounded space charge field \( E_{sc} \) with applied external field \( E_{ext} \). Also, it is nonlocal, refractive index change depends in each point on the intensity distribution in the whole transverse plane. Self-focusing nonlinearity is determined with \( E_{ext} > 0 \), in contrast to defocusing nonlinearity where \( E_{ext} < 0 \). Linear and nonlinear light propagation effects in photorefractive SBN crystal throughout this thesis are explained by the basic propagation model which includes joined propagation (EQ. (5.1.11)) and potential (EQ. (5.1.12) or EQ. (5.1.13)) equations.

This model is used for examination of linear and nonlinear propagation of paraxial light beam with initial intensity distribution defined by Mathieu beam (single or elliptic Mathieu beam) in nonlinear photorefractive SBN crystal. The strength of nonlinearity is varied by the strength of the external electric field \( E_{ext} \) (typically \( E_{ext} = 0.8 - 2 \) kV/cm) or intensity \( I \) of certain Mathieu beam. This model allows simulation of experimental realization of Mathieu lattices in photorefractive SBN crystal, biased by the external electric field \( E_{ext} \) along the crystal’s optical \( c \)-axis (as shown in Fig. 4.2). As a writing beam for lattice is used ordinary polarized Mathieu beam. According to this writing beam propagates in linear regime inside the crystal, and due to refractive index modulation creating 1D or 2D Mathieu photonic lattices.

In some cases, the two-dimensional nonlinear paraxial Schrödinger equation can be solved analytically, but according to the requirements in this thesis, comprehensive numerical model have to be implemented to solve this equation and model the light propagation in nonlinear media. One of the methods to numerically solve propagation (EQ. (5.1.11)) and potential (EQ. (5.1.12) or (5.1.13)) equations is a symmetrized split step propagation method [81]. First, the model equations are rewrite in dimensionless form, introducing the dimensionless variables: \( X = x/x_0, Y = y/x_0, Z = z/k_z x_0^2 \) and \( \Phi_{sc} = \phi_{sc}/x_0 E_{ext}, (x_0 \) is a transverse scaling factor)
\[ i \frac{\partial U}{\partial Z} + \frac{1}{2k_z}[\nabla_\perp^2 + V(I)]U = 0, \]
\[ \Delta \Phi_{sc} + \nabla \ln(1 + I) \nabla \Phi_{sc} = \frac{\partial}{\partial z} \ln(1 + I) \]

with \( \nabla_\perp^2 = \left( \frac{\partial}{\partial X^2} + \frac{\partial}{\partial Y^2} \right) \), where \( V(I) = \frac{k_z^2 x_0^2}{n_0^2} \Delta n^2(I) = -k_z^2 (n_0^{oe})^4 r_{13} E_{sc} \) is potential optically induced in nonlinear phorefractive crystal \((n_0^{oe}) - \) ordinary or extraordinary refractive index with corresponding linear electro-optic coefficient \( r_{13} \) or \( r_{33} \), respectively).

### 5.2 Propagation of light in photonic lattices

To propagation of light in optically induced photonic lattice into photorefractive SBN crystal is examined by the nonlinear Schrödinger equation for an initial paraxial scalar light field \( U(\mathbf{r}) \) with longitudinal wavevector \( k_z \)

\[ i \frac{\partial U(\mathbf{r})}{\partial Z} + \frac{1}{2k_z}[\nabla_\perp^2 + V(I)]U(\mathbf{r}) = 0. \]  

Nonlinear potential depends on the incident light intensity, defined by photorefractive nonlinearity

\[ \Delta \Phi_{sc} + \nabla \ln(1 + I + I_{latt}) \nabla \Phi_{sc} = E_{ext} \frac{\partial}{\partial x} \ln(1 + I + I_{latt}), \]  

and \( I_{latt} \) denotes the lattice intensity distribution. Propagation (5.2.15) and potential (5.2.16) equations are related and for their numerically solution is used symmetrized split-step beam propagation method [81]. Again, the model equations are rewrites in dimensionless form, introducing the dimensionless variables: \( X = x/x_0, Y = y/x_0, Z = z/k_z x_0^2 \) and \( \Phi_{sc} = \phi_{sc}/x_0 E_{ext}, \) \( (x_0) \) is a transverse scaling factor

\[ i \frac{\partial U}{\partial Z} + \frac{1}{2k_z}[\nabla_\perp^2 + V(I)]U = 0, \]
\[ \Delta \Phi_{sc} + \nabla \ln(1 + I + I_{latt}) \nabla \Phi_{sc} = \frac{\partial}{\partial x} \ln(1 + I + I_{latt}) \]

This model is used for examination of linear and nonlinear effects of probe beam propagation (plane wave, elliptic optical vortex, or Gaussian beam) in different Mathieu photonic lattices. One possibility is to use a single Mathieu beam to create photonic lattices. Furthermore, numerous aperiodic lattices are formed via the interference of Mathieu beams.
5.3 The symmetrized split-step beam propagation method and calculation of Potential equation

In past, various numerical methods are revealed to calculate NPE. Due to the problem a convenient numerical method is chosen. The symmetrized split-step beam propagation method (Symmetrized SSBM) or the symmetrized split-step Fourier method is one technique for numerical solving NPE and potential equations. Symmetrized SSBM has been revealed as a rather fast and reliable method, appropriate for problems inside this thesis.

Primarily, NPE and potential equation are written in dimensionless form like EQ. (5.1.14) or EQ. (5.2.17). The dimensionless NPE is separated in terms of dispersion and nonlinearity. To use Symmetrized SSBM for solving NPE dispersion and nonlinearity terms are decoupled for a small propagation distance, \( \Delta Z \). Operators \( \hat{D} \) and \( \hat{N} \) are written to correspond to the dispersion and nonlinearity terms, respectively. Operator form of NPE is

\[
\frac{\partial U(r_\perp, Z)}{\partial Z} = i(\hat{D} + \hat{N})U(r_\perp, Z)
\]

(5.3.18)

with dispersion \( \hat{D} = \frac{1}{k_z} \nabla_\perp^2 \) and nonlinear operator \( \hat{N} = V(I) \).

Whole propagation length is divided into small spatial steps, with step size \( \Delta Z \). To avoid errors proper choice of step size and optimal computational window in the transverse plane \( (X, Y) \) are required. Propagation in one step, from \( Z \) to \( Z + \Delta Z \), is calculated in alternating steps in which either diffraction effect or nonlinear effect are considered. A formal solution of equation EQ. (5.3.18) for the propagation from \( Z \) to \( Z + \Delta Z \) is given as

\[
U(r_\perp, Z + \Delta Z) = e^{\Delta Z(\hat{D} + \hat{N})}U(r_\perp, Z),
\]

(5.3.19)

where \( r_\perp = (X, Y) \) yields transverse coordinates.

The numerical model is derived by applying the Baker-Hausdorff formula for noncommutative operators \( \hat{D} \) and \( \hat{N} \) [82]. For Symmetrized SSBM, first is computed diffraction effect over the half step size, \( \Delta Z/2 \), then is computed nonlinearity effect at the step midpoint for the whole step, and in the end diffraction effect is computed over \( \Delta Z/2 \).

Diffraction is obtained by using a pseudospectral method, which includes the Fast Fourier Transform (FFT) and the nonlinear effect is neglected. FFT of the envelope at the propagation distance \( Z \) facilitates computation of the differential operator by \( \nabla_\perp^2 = -(k_x^2 + k_y^2) \). An inverse Fourier transform then gives the diffracted field envelope at the propagation distance \( Z + \Delta Z \).

The nonlinear correction depends on incident intensity distribution. To implement nonlinear correction over the whole step \( \Delta Z \) first the potential equation is calculated in over propagation step by an iterative procedure in order to potential dependence on initial intensity, afterwards propagation of the light field is calculated for the whole step including potential.

To compute the propagation four Fourier Transforms are used in one step via Symmetrized SSBM. The computation of one step of the propagation can be summarized as (schema represented in Fig. 5.1)

\[
U(r_\perp, j\Delta z) = \mathcal{F}^{-1} \left[ \exp(\frac{\Delta z}{2} \hat{D}(i\omega)) \mathcal{F} \left[ \exp\left( \int_{z}^{z+\Delta z} \hat{N}(z')dz' \right) \right] \right] \mathcal{F}^{-1} \left[ \exp(\frac{\Delta z}{2} \hat{D}(i\omega)) \mathcal{F} \left[ U(r_\perp, (j-1)\Delta z) \right] \right]
\]

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Nonlinear paraxial and potential equations are coupled in problems involved in this thesis. The potential equation provides information about the photorefractive nonlinearity during propagation, determined by initial intensity distribution, $E_{sc}(I)$. For numerical simulation of such equations model, the iterative calculation procedure is applied. In every iteration step, the Symmetrized SSBM is applied with determined refractive index modulation and light field distribution at the initial position. Hence, this is an initial value problem, which makes it computationally efficient.
Chapter 6

Results and discussion

6.1 Nonlinear self-action of Mathieu beams in SBN crystal and Mathieu photonic lattices

In this section nonlinear self-action of single and elliptic Mathieu beams in photorefractive SBN crystal is explored with increasing nonlinearity strength. They would be exploited as writing light for fabricating discrete nonlinear photonic waveguide structures or dynamical chiral photonic structures with a tunable chirality.

6.1.1 Switching discrete diffraction in nonlinear Mathieu lattices

Mathieu beams, as nondiffracting beams, have the propagation-invariant intensity distributions in a vacuum. However, the question is does they remain nondiffracting in the nonlinear photosensitive medium? Thus, linear and nonlinear propagation of single Mathieu beams would be investigated experimentally and numerically in the photorefractive SBN crystal. The experimental setup for this investigation is depicted in Fig. 4.3.

In the beginning, the free-space propagation of even Mathieu Beam of order zero, with ellipticity \( q = 25 \) and characteristic structure size \( a = 25 \mu m \) is examined. The transverse intensity distribution is shown in Fig. 6.1 (A1) has the quasi 1D discrete intensity distribution, with corresponding phase distribution Fig. 6.1 (A2). In free-space it propagates invariant over a 6.36mm, which corresponds to 15mm in the homogenous SBN crystal as shown in Fig. 6.1 (A3). The same conclusions are worthy of higher-order Mathieu beams.
Fig. 6.2 (A) shows that the zeroth-order Mathieu beam has one path which corresponds to 1D discrete intensity distribution. Hence, the zeroth-order Mathieu beam is appropriate for the examination of nonlinear self-action in the 1D path. If the Mathieu beam order is incremented the number of paths increases as shown in Figs. 6.2 (B), (C), (D), but now straight and curved paths are noticeable. The transverse intensity distribution of higher-order Mathieu beams consist of multiple paths, which can serve as 2D photonic lattices. Thus, the dimensional crossover from 1D to 2D photonic lattices is apparent from the zeroth-order Mathieu beam to fifteenth-order Mathieu beams. The higher order Mathieu beams are appropriate for an examination of nonlinear self-action in curved 2D paths. One interesting property of higher-order Mathieu beams is that while the order increase inner paths are straight with equidistant intensity spots. By changing order or ellipticity, different distances between spots are achievable. Due to this, higher-order Mathieu beams are convenient for facile realization od periodic patterns, reducing realization time.

Mathieu beams are suited for the investigation of switching diffraction in self-induced waveguides, experimentally and numerically. The lattice-fabricating zeroth-order Mathieu beam depicts in Fig.6.1 shows nonlinear discrete diffraction as a result of self-action in dependence of the beam power $P$ that influences the strength of the nonlinearity, shown in Fig. 6.3. In the first column (Figs. 6.3 (A1) - (A3)) are represented simulated $yz$ cross-sections through the intensity volume, in the second one (Figs. 6.3 (B1) - (B3)) the simulated transverse intensity distributions, while in the third one (Figs. 6.3 (C1) - (C3)) the experimentally obtained transverse intensity distributions at the back face of the SBN crystal.

For lowest power $P = P_0 = 10\mu W$ the waveguides are well fabricated as presented in the
The noticed discrete diffraction of Mathieu beam is like as broad Gaussian beam propagation in 1D periodic waveguide arrays [27]. For doubled beam powers \( P = 2P_0 \) and \( P = 4P_0 \) spreading of the highest intensities are observed away from the center and towards the outer parts along the \( y \)-axis as shown in Figs. 6.3 ((A2) - (C2)) and (A3) - (C3)). Considering that the initial zeroth-order Mathieu beam has its maximum in the origin and the envelope of the 1D intensity distribution is along the \( y \)-axis the refractive index modulation and thus the self-action of the writing beam is strongest in the center. For high beam power of \( 4P_0 \) thermal diffusive effects are increased along the optical \( c \)-axis, parallel to the \( x \)-axis, noticeable along the \( x \)-axis as the shift in intensity in (B3) and (C3). According to this investigation, the new effect is founded, named \textit{1D nonlinear discrete diffraction} or \textit{morphing discrete diffraction}. This effect is a nonlinear pandan of linear discrete diffraction in 1D waveguide arrays.

![Image](attachment.png)

Figure 6.3: Nonlinear discrete diffraction of zeroth-order Mathieu beam in dependence of the strength of the nonlinearity controlled by beam power \( P \). First column presents simulated cross section through the volume at the orientation indicate with dashed line in (B1) for increasing beam powers. Second and third column presents simulated and experimentally observed intensity distributions at the back face of SBN 15mm long crystal. \( P_0 = 10 \mu W \).

Also, higher order Mathieu beams with ellipticity \( q = 325 \) and characteristic structure size \( a = 25 \mu m \) were investigated with increasing beam powers \( P \). Figure 6.4 shows the result of nonlinear self-action of sixth-order Mathieu beam. First row (A) shows simulated and second row (B) experimentally observed transverse intensity distributions at the back face of the 15mm long SBN crystal in the according to a successive doubling of the initial beam power \( P_0 = 10 \mu W \). Intensity profiles along each hyperbolic layer depicted in (B) are shown in treed row (C).

For the lowest power \( P = P_0 \), the highest intensities, located in the center of an initial sixth-order Mathieu beam, are redistributed towards the outer parts in the \( y \)-direction, which confirms by the intensity profile along the green line Figs. 6.4 (B1) and (C1). Only central hyperbolic arms of the Mathieu lattice feel the lowest nonlinearity, also visible on the intensity profile along further hyperbolic layers. When nonlinearity strength is incremented by doubling the power \( P = 2P_0 \)
and $P = 4P_0$, even further hyperbolic arms of the Mathieu lattice are affected. Central intensities spread outwards along 2D curves (Figs. 6.4 ((A2), (B2)) and ((A3), (B3))), confirmed and by intensity profiles (Figs. 6.4 ((C2), (C2))). Thermal diffusion additionally introduces the shift in the $x$-direction, intensities merging due to modulation instabilities influence the intensity redistribution and both experimental and numerical results are asymmetric. Still the pure effect of 2D nonlinear discrete diffraction along hyperbolic paths is in general recognizable. When the diffusive term in potential equation is neglected, numerically simulated intensity distribution would be symmetric (not shown).

Figure 6.4: Switching discrete diffraction on curved paths based on the self-action of sixth-order even Mathieu beam: (A) simulated and (B) experimentally observed transverse intensity distributions at the crystal’s back face. (C) Intensity profiles along the hyperbolic waveguide layers indicated in (B). $P_0 = 10\mu W$.

Figure 6.5 shows the result of nonlinear self-action of the eleventh-order Mathieu beam. The first row shows simulated and second row experimentally observed transverse intensity distributions
at the back face of the 15mm long SBN crystal in the dependency of a successive increase of the initial beam power $P_0 = 20\mu W$. For the lowest power, the only central arm of eleventh-order Mathieu beam is slightly affected by nonlinearity and the highest intensity located in the center is diffracted to out parts in the $y$-direction (Figs. 6.5 (A1), (A2)). But, with power increasing additional hyperbolic arms are affected and central intensities are spreading outwards along 2D curves, depicted in Fig. 6.5 (B1), (B2), (C1), (C2). For higher power thermal diffusion shift in $x$-direction appears, creating modulation instabilities, merging of the intensities visible as asymmetric intensity distributions.

Figure 6.5: Switching discrete diffraction on curved paths based on the self-action of eleventh-order even Mathieu beam: first row simulated and second experimentally observed transverse intensity distributions at the crystal’s back face. $P_0 = 20\mu W$.

Once more, intensity distributions that reflect linear discrete diffraction are observed but in the nonlinear regime and the outward-directed intensity transport in the nonlinear lattices follows mainly along with each hyperbolic layer of the higher order Mathieu beam. In this examination switching discrete diffraction along curved 2D paths is realized.

Figure 6.6: Narrow Gaussian probe beam in Mathieu lattice potentials from: (A) Fig. 6.3 (A1); (B) Fig. 6.4 (A1) central waveguides; and (C) Fig. 6.4 (A1) purple waveguide layer.
Linear propagation of narrow Gaussian beams in Mathieu lattices presented in the previous paragraphs are numerically simulated and presented in Fig. 6.6. Discrete Mathieu lattices by themselves imprint the intensity distribution on probing light beam that is typical for discrete diffraction. Figure 6.6 (A) images the intensity distribution of such a Gaussian probe beam inside the Mathieu lattice shown in Fig. 6.3 (A1). The initial plane in Fig. 6.6 marks the lattice. The perpendicularly launched probe beam flows from one waveguide to another, generating diffraction characteristics as in 1D waveguide arrays. Next, the probe beam is launched in the central spot of the central arm of the 2D lattice shown in Fig. 6.4 (A1). The intensity distribution of the probe beam inside the lattice is depicted in Fig. 6.6 (B) showing that probe beam diffraction characteristics as discrete diffraction in 2D lattices. In the end, probe beam is launched in a central spot of an outer hyperbolic layer imaged in Fig. 6.4 (A1). The intensity distribution of the probe beam inside the lattice is depicted in Fig. 6.6 (C), demonstrating 1D discrete diffraction along the hyperbolic waveguide layer.

This subsection presents nonlinear self-action in Mathieu beams leading to switching discrete diffraction. Mathieu beams of different orders are investigated in nonlinear photorefractive SBN crystal, numerical and experimentally. First, linear discrete diffraction is connected with nonlinear self-effects in the quasi one-dimensional lattice. Then the same effect is observed in the two-dimensional lattice with a gradual transition from one to two dimensions. The term switching diffraction is used to explain nonlinear behavior similar to discrete diffraction, phenomena characteristic for linear propagation of light in periodic arrays or lattices.

6.1.2 The self-action of elliptical Mathieu beams in nonlinear media

The self-action of elliptical Mathieu beams in nonlinear 20mm long SBN crystal is investigated experimentally and numerically as well as their orbital angular momentum. Single scalar even and odd Mathieu beams exhibit only real-values field distribution and their transverse Poynting vector therefore vanishes. Elliptic Mathieu beams are the complex superposition of even and odd Mathieu beam mathematically described with EQ. (3.2.8). In contrast to single Mathieu, their intensity distributions are distinguished by a series of concentric ellipses, while the phase distributions are continuously modulated along that ellipses and they possess transverse energy flow [69].

Generally, the energy flow of light is determined by both, its spin angular momentum and its orbital angular momentum (OAM), which is described by the Poynting vector [83]. For linear propagation of continuously modulated nondiffracting beams in homogeneous media, the energy flow is hidden because the transverse intensity distribution stays invariant and the energy flow is continuously redistributed. Linearly polarized, the transverse light field has the transverse time-averaged Poynting vector \( \langle S \rangle \) determined by the spatial OAM distribution and given by [84]

\[
\langle S \rangle = \frac{i \omega \epsilon_0}{2} (Ely^* \nabla Ely - Ely \nabla Ely^*),
\]

where \( \omega = ck \) is the angular-frequency that connects the speed of light \( c \) with the wave number \( k = 2\pi/\lambda \), defined by the wavelength \( \lambda \). \( \epsilon_0 \) is the vacuum permittivity, and \( Ely \) denotes elliptic Mathieu beam. Here is considered only the transverse \( (x, y) \) component of the Poynting vector.
Figure 6.7 depicts the intensity and phase distribution of an elliptic Mathieu beam of order $m = 10$ with an ellipticity $q = 25$ and characteristic structure size $a = 25 \mu m$ at the initial plane of the crystal ((A), (B)). Numerical and experimental transverse Poynting vectors are calculated with EQ. (6.1.1), and corresponding Pointing vectors are indicated in Fig. 6.7 with overlying arrows. Also, Fig. 6.7 (C) presents numerically simulated visualization intensity distribution of the such Mathieu beam through the 2mm long crystal.

In numerical simulations, the electric field, $\psi = E ly$ is calculated and by using relation $\psi = I e^{i\phi}$, intensity ($I$) and phase ($\phi$) distributions are obtained as shown in the first row in Fig. 6.7 (A1), (A2)). Such calculated electric field is used for numerical Poynting vector calculation of the elliptic Mathieu beams according to EQ. (6.1.1). In contrast, in the experimental realization, only the transverse intensity ($I$) and phase ($\phi$) distributions are accessible (Fig. 6.7 (B1, B2)). Using relation $\psi = I e^{i\phi}$ the experimentally electric field is obtained and used for calculation of experimental Poynting vector of the elliptic Mathieu beam toward to EQ. (6.1.1).

Linear propagation of elliptic Mathieu beams in homogeneous media has balanced intensity redistribution (Fig. 6.7 (C)). In the following, elliptic Mathieu beams are examined in nonlinear regimes. The results are demonstrated for the elliptic Mathieu beam of order $m = 10$ (Fig. 6.7) but the same conclusions would be demonstrated for different parameters of elliptic Mathieu beams. Nonlinear propagation of elliptic Mathieu beams in SBN crystal is investigated with the experimental setup shown in Fig. 4.3, and corresponding transverse intensity distributions from the back face of the SBN crystal are imaged. Experimental results are compared with matching numerical simulations.
The investigation starts with the nonlinear self-action of the elliptic Mathieu beam with a structure size of \( a = 15 \mu m \) in the SBN crystal. Refractive index modulation is optically induced with elliptic Mathieu beam as a writing beam. Such induced refractive index depth estimates in the order of \( 10^{-4} \), both experimentally and numerically. The initial beam power \( P_0 \approx 20 \mu W \) is increased twice, by doubling in two steps, both in numerical simulations and experiments. The results are shown in Fig. 6.8, where the first row represents the transverse intensity distributions at the back face of the SBN crystal from numerical simulations while the second row shows corresponding experimental results. For a demonstration of energy flow of elliptic Mathieu beam in SBN crystal, only the numerical calculated Poynting vector is observed and they are indicated with arrows in Figs. 6.8.

At the front face of the crystal, the Poynting vector is along the initial ellipse (Fig. 6.7). For low power \( P = P_0 \) the Poynting vector stays directed along the initial ellipse even at the back face of the crystal after nonlinear self-action as depicted in Fig. 6.8 (A1). Numerically and experimentally observed intensity distributions at the back face shows a high agreement. It is shown that elliptic Mathieu beam propagates almost linearly for low power and output intensity distribution is almost unchanged, so the beam is still nondiffracting.

Nonlinear self-interaction is gradually increased by doubling the power of \( P = 2P_0 \), due to that the intensity distribution is changed and the breaking of the energy flow is demonstrated. Beforehand smoothly ellipse is modulated in the form of occurring accumulations of intensity, and the writing beam thus can not be considered as nondiffracting Mathieu beam. For the highest beam power of \( 4P_0 \), the ellipse is broken in separated spots of high intensity. These high spots intensity rotate in the direction indicated by the Poynting vector (Fig. 6.8 (A3)). The number of spots depends on the
order $m$ of the elliptic Mathieu beams, but the strength of nonlinearity or the propagation distance also changes the number of high intensity spots.

The spots that emerged at the back face of crystal are a consequence of modulation instabilities on an ellipse [85]. The anisotropic medium and the modulation of the intensity distribution, which mostly occurs along the intrinsic ellipse, establish the refractive index modulation in the direction of the optical $c = x$ -axis. The energy flow, along the intrinsic ellipse, is directed perpendicular to the optical axis where the refractive index modulation is weak, while the flow parallel to the $c$-axis is hindered because the refractive index modulation is strong. The conglomerations of high-intensity appear in particular at the trough of the high refractive index thus enough intensity is accumulated to create solitary strands of increased refractive index. These solitary strands of the increased refractive index form a twisted photonic structure inside the photorefractive crystal.

Experimental intensity distribution shows asymmetry due to the thermal diffusion effect along the optical $c$-axis. But, the numerical intensity distributions are simulated with neglected diffusion part in potential equation (EG. (5.1.12)).

Numerical simulations are used for illustration of how the main intensity, distributed on an ellipse in the front face, propagate through the nonlinear 20mm long SBN crystal. In Fig. 6.9 (A) is presented the 3D distribution of the main intensity inside the crystal for the photonic structure depicted in Fig. 6.8 (A3). It is visible that after some propagation distance the main intensity is separated into high intensity spots, which rotates in the direction determined by energy flow, thereby, rotating refractive index strands are forming.

Figure 6.9: Numerical simulation of the 3D intensity volume inside 20mm long SBN crystal. Fabrication of photonic structure (A) from Fig. 6.8 (A3) and (B) from Fig. 6.10 (A3).

The investigation is dedicated to how characteristic structure size $a = 2\pi/k_t$ of elliptic Mathieu beam influence on intensity filamentations in the nonlinear medium. Figure 6.10 shows the influence of increasing characteristic beam sizes $a = [15, 20, 25] \, \mu m$ to the nonlinear propagation for constant beam power of $P_0 \approx 20\mu W$, experimentally and numerically. Arrows again indicate the numerically calculated Poynting vector. For $a = 15\mu m$ (Fig.6.10 (A1)) conglomerations of intensity is not visible, but for $a = 20\mu m$ filamentation occurs as depict in Fig.6.10 (A2). This filamentation is similar to one occur for $a = 15\mu m$ with the higher power $P$ in between $2P_0$ and $4P_0$ depicted in Figure 6.8.
For structure size $a = 25\mu$m the self-action is strong and tends to become stronger for larger structure sizes $a$. In contrast to previous cases, the intensity spreads to ellipses located more outside due to increasing modulation instabilities and more spots with high intensity are observed. Consequently, the Poynting vector directs outward for the outer high-intensity spots (Fig.6.10 (A3)). Experimental results are asymmetric due to the thermal diffusion effect, which is neglected in numerical calculation.

For this structure size 3D distribution of the main intensity inside the crystal is presented in Fig. 6.9 (B). After some propagation distance, the main and next intensities are separated into high intensity spots, which rotates in the direction determined by energy flow generating rotating refractive index strands. It is demonstrated that by increasing the structure size $a$ of elliptic Mathieu beams, the local slope of the helix of the emerging rotating strands of higher refractive index is decreased.

Figure 6.9 presents the numerical visualization of rotating high-intensity filaments, i.e. 2D twisted waveguides through the crystal due to the nonlinear self-interaction of elliptic Mathieu beams. An enhanced degree of branching could be observed in the dependency of beam size $a$ of elliptic Mathieu beam. By changing beam size $a$ and power $P$ it is possible to manage the number of rotating strands and their slope. This illustrates optically induced chiral Mathieu photonic lattice where the rotation of waveguides is directed by the direction of the energy flow of elliptic Mathieu beams, with the opportunity to change the period of rotation as well as the radius of waveguides. Further investigations could potentially show advanced light-matter interactions, e.g., when probing these diverse chiral structures with chiral light.

This section represents an approach to identify and visualize the energy flow of light based
on the symmetry breaking by nonlinear light-matter interaction of OAM carrying beams. Elliptic Mathieu beams with outstanding continuously modulated OAM distributions are used for the purpose. It is revealed that the nonlinear self-action of elliptic Mathieu beams managed the formation of high-intensity filaments, which rotated in the direction determined by the energy flow. It is examined how the strength of the nonlinearity and the structure size of the Mathieu beams influence the emerging photonic structure. Twisted refractive index formations, which could act as chiral waveguides, are observed in the limited regime with proper parameters of the nonlinearity and structure size. Hence, by this approach, it is provided a new method for the realization of the chiral photonic lattices with longitudinally increasing "helix slopes", and additionally tailored transverse ellipticities.
6.2 Elliptical vortex necklaces in Mathieu lattices

In this section, linear and nonlinear excitation of two-dimensional Mathieu photonic lattices induces in photorefractive SBN crystal is investigated experimentally and numerically. Elliptical vortex beam is used as probe beam. First, a single Mathieu beam is used to fabricate Mathieu lattices and the propagation of elliptic vortices is examined in them. The main goal of this examination is to provide the conditions for the existence of spatially localized vortex stats.

The optical vortex possesses a phase singularity and a rotational flow around the singular point in a given direction can be applied in many physical systems [86]. Different single Mathieu beams are used as writing light for the optical induction of Mathieu lattices with different shapes. Certain Mathieu lattices optically induced in nonlinear SBN crystal are an exemplary two-dimensional photonic structure for the examination of the propagation of elliptical vortex beams.

The experimental setup for this investigation is depicted in Fig. 4.4. The experiment is realized in photorefractive SBN crystal with a geometrical dimension 5x5x15mm³. Nd: YVO₄ laser is used as the light source. Crystal is externally biased with an electric field $E_{ext} = 1600$ V/cm aligned along the optical $c = x$-axis, perpendicular to the direction of propagation ($z$-axis). As for writing beam, even Mathieu beams of order $m = 8$, characteristic beam size $a = 90\mu m$, and various ellipticity parameters $q$ is used, and the elliptic vortex is used as the probe beam, shown in Fig. 6.11.

This investigation starts by considering the Mathieu lattices optically induced with even Mathieu beam of order $m = 8$, with increasing ellipticity parameter $q$. The ellipticity parameter $q$ is gradually increased allowing change of the photonic structure shape from a circle to an ellipse. The presence of the lattice during vortex propagation induces separation of confinement elliptic vortex in filaments around the location of the incident vortex ring and the surrounding lattice sites.

![Figure 6.11: Characteristic of Mathieu lattice and elliptic vortex. (A) Intensity distribution for Mathieu lattice of order $m = 8$, ellipticity $q = 15$ and structure size $a = 90\mu m$ in the front face of the SBN crystal. (B), (C) Intensity and phase distributions of elliptic vortex in the front face of the SBN crystal.](image-url)
Figure 6.12 summarizes results for three different values for ellipticity parameter $q$ of Mathieu lattices. The input vortex beam with topological charge $C_T = 1$ varies to cover the sites on the inner lattice ring. The first two columns show results from numerical simulations while in the third one experimentally observed results are presented. In the case with no ellipticity ($q = 0$), a stable necklace beam is observed for a quasi-linear case (very low nonlinearity) (Fig. 6.12 (A)). Next, the lattice ellipticity is increased while other parameters are unchanged. Again elliptical necklaces are obtained but necklace "pearls" are slightly close to each other, determined by shape and distribution of the lattice sites in that lattice area. These vortex states are stable during propagation inside the 15mm long crystal.

Table:

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Figure 6.12: Elliptical necklaces in Mathieu lattices with different ellipticity parameter $q$ and topological charge $C_T = 1$. The input vortex beam is shown with the layout of the lattice beams indicated by open circles (the first column). The corresponding intensity distributions are shown at the exit crystal face in numerical results (second column) and experiment (third column). Numerical lattice intensity $I_{latt} = 0.3$, and input vortex intensity $I = 0.005$; the experimental lattice power $P_{latt} = 20 \mu W$ and input vortex power $P = 8 \mu W$. 

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When beam order \((m > 8)\) is increased, elliptical necklaces with a larger number of pearls are observed. These structures are stable along with propagation for the length of the crystal and topological charge \(C_T = 1\). Figure 6.13 presents the intensity distributions of elliptical necklaces in Mathieu lattices of order \(m = [9, 10, 11, 12]\) at the exit crystal face from numerical simulations. If the broader vortex beam covers more rings of the Mathieu lattices multiple necklaces are observed but they are not stable during propagation (Fig. 6.14).

![Figure 6.13: Stable elliptical necklaces in Mathieu lattices of different order \(m\) with ellipticity parameter \(q = 0\) and topological charge \(C_T = 1\): (A) \(m = 9\), (B) \(m = 10\), (C) \(m = 11\), (D) \(m = 12\).](image)

In further, higher-order vortex beams are investigated in Mathieu lattices with \(q = 15\). As the probe beam is used the same vortex beam but with different topological charge \(C_T\). The results are depicted in Fig. 6.15. While \(C_T\) increment, energy flow inside the inner lattice ring causes an increase in asymmetry. But from the overall phase distribution, it is noticeable that the central area still preserves the expected vortex state. The phase on the inner lattice ring still corresponds to the input \(C_T\), but it is circularly shifted with respect to the central vortex area. Phase distributions along the propagation are shifted along the inner lattice ring, as well as in the central part of the phase distributions.

![Figure 6.14: Multiple elliptical necklaces in Mathieu lattices of order \(m = 8\) with ellipticity parameter \(q = 0\) and topological charge \(C_T = 1\): (A) The input vortex beam is shown with the layout of the lattice beams indicated by open circles. Numerically observed (B) phase and, (C) intensity distributions at the back face of the crystal. Numerical lattice intensity \(I_{latt} = 0.3\), and input vortex intensity \(I = 0.005\).](image)
In contrast to the conventional multiple-charged vortex where the embedded phase singularity is multiply folded, the elliptical necklaces show an unfolded behavior in the phase distribution. For higher-order elliptical vortices, a spatial separation of several single-charged phase singularities is observed [87]. Phase singularities separation has the finite distance and depends on the lattice ellipticity, or the topological charge of input vortex $C_T$. The Euclidean distance between the two furthest singularities, as indicated in Fig. 6.15 (B2), are calculated for different lattice ellipticity and the results are presented in Fig. 6.16 (A). For higher ellipticities, $q$ higher values of separations distance are observed. When $C_T$ increase, phase singularity separation distances also increases for all ellipticities. Phase singularity separation distances are also calculated during propagation, for higher $C_T = 2, 3, 4$ and $q = 15$ (Fig. 6.16 (B)). As propagation distance increase higher values of separations are observed for higher $C_T$. 

Figure 6.15: Single- and multiple-charged elliptical necklaces. (A1) - (D1) Numerically observed intensity and (A2) - (D2) phase distributions. (A3) - (D3) experimentally observed intensity distributions at the back face of the crystal. The lattice ellipticity $q = 15$ and other parameters are as in Fig. 6.15.
Figure 6.16: Phase singularity separation. (A) Phase singularity separation versus $C_T$ for various lattice ellipticity after 15mm propagation distance. (B) Phase singularity separation versus propagation distance for various $C_T$ for $q = 15$ (Fig. 6.15). Separations are measured between the two singularities for lattice ellipticity as a Euclidean distance.

The elliptical necklaces are obtained in the quasi-linear regime. Their stability is investigated for propagation distances longer than crystal size via numerical simulations. It is established that elliptical necklaces are stable for propagation length of few crystal sizes, but they transform to oscillating dipole states after 10cm propagation length, as shown in Fig. 6.17 (A)-(C). Their central part of phase distribution remains unchanged, in contrast along the inner lattice ring where initial phase distribution is completely broken. Higher-order vortex states shown in Fig. 6.15 (B)-(D) are slightly asymmetric and stable only for short propagation distance comparable with crystal size.

The standard definition for the (normalized) $z$ component of the orbital angular momentum (AM) gave as [88]

$$L_z = -\frac{i}{2} \int \int dxdy A^*(x,y)(x\partial_y - y\partial_x)A(x,y) + cc.$$  \hspace{1cm} (6.2.2)

is used to calculate the orbital AM of necklace beams $A(x,y)$ during propagation. Figure 6.17 (D) presents the mean orbital AM, $L_z$ per transverse plane dependent on the propagation distance $z$ of the necklace states along the propagation distance for different ellipticity parameters $q$. For higher lattice ellipticity, AM transfer is less in compression for lower ellipticities where the neighboring lobes exchange more power during the propagation (depicted with a red plot in Fig. 6.17 (D)).

**Figure 6.17:** Dipole states in Mathieu lattices of various ellipticity (A) $q = 0$, (B) $q = 10$, and (C) $q = 15$. Intensity and phase distributions are presented after 10cm propagation. (D) Normalized $z$ component of the angular momentum along the propagation distance. Other parameters are as in Fig. 6.12.
In the end vortex stability with increasing nonlinearity is investigated. Experimental and numerical results are presented in Fig. 6.18 for vortex states with $C_T = 1$ and lattice ellipticity $q = 15$ for 15mm long crystal. With lower powers, neighboring lobes exchange some power, the elliptical necklace is damaged, and regular oscillations along the propagation are found [Fig. 6.18 (A)]. For higher beam power, irregular oscillations are observed, even more noticeable for longer propagation distances [Fig. 6.18 (B)]. From phase distribution, it is visible that only the central part stays unchanged, in contrast to the inner lattice ring.

Figure 6.18: Nonlinear vortex propagation in Mathieu lattices. (A1), (B1) intensity distributions, (A2), (B2) corresponding phase distributions at the exit face of crystal obtained from numerical simulations, (A3), (B3) intensity distributions at the exit face of crystal experimentally obtained. Input vortex intensities in numerical simulation are (A1) $I=0.01$ and (B1) $I = 0.1$, with appropriate input vortex power in experiment (A3) $P = 20 \mu W$ and (B3) $P = 30 \mu W$.

In this section, experimental and numerical investigation of the elliptical vortex inside the optically induced Mathieu lattices in 15mm long SBN crystal is presented. Stable elliptic necklaces are obtained, control of the shape and size of the elliptical necklace was analyzed, as well as the number of pearls in them by changing order of the Mathieu lattices and their ellipticities. For higher-order vortices, it has been noticed separation of phase singularity, which is calculated. It is demonstrated that separation increase with increasing $C_T$, ellipticity or propagation distance. The conditions for stable elliptic necklaces were found and their orbital AM is measured. Oscillating dipole states or dynamic instabilities were observed for longer propagation distances, higher beam power, and higher-order vortices. These results enable further investigations of vortex beam control in photonic lattices optically induced by other than Mathieu beams and they have potential applications in the field of optical micromanipulation to guide, trap, and sort objects.
6.3 Creation of aperiodic photonic lattices and propagation of light in the...

In this section, would be considered an influential approach for the creation of the two-dimensional (2D) aperiodic photonic lattices in a SBN crystal by using Mathieu nondiffracting beams as well as the linear and nonlinear propagation of light in such formed lattices.

6.3.1 Creating aperiodic photonic structures by synthesized Mathieu-Gauss beams

Even Mathieu beams have different intensity distributions as shown in Fig. 3.3, hence it is possible to create different 2D photonic lattices only by changing the order \( m \), or ellipticity \( q \) of a single Mathieu beam. Such photonic lattices are well-suited systems for control and manipulation of light propagation. However, in this section 2D aperiodic photonic lattices are created by synthesizing two or more even Mathieu beams. Even Mathieu beams with the same ellipticity \( q = 25 \) and characteristic beams size \( a = 25 \mu m \) are used. Additionally, the Mathieu beams are apodized with Gaussian beams which yield to finite energy pseudo-nondiffracting beams, Mathieu - Gauss beams (MG) [89].

Three different ideas presented in this section for the creation of various complex aperiodic Mathieu beams would be used for optically-induced photonic lattices. These results will open new future research especially in the field of light propagation in such aperiodic photonic lattices and potential application in the realization of optical devices.

The interference of two even MG beams of a different order \( m_1 \) and \( m_2 \) is investigated and their transversal intensity distribution is reproduced from numerical simulations and experiment in Fig.6.19. When MG beams with the same orders parity in phase configuration \((m_1, m_2)\) - both even or both odd - interfere the observed structures are symmetric according to \( y \)-axis. Two examples for \( m_1 = 0, m_2 = 10 \) and \( m_1 = 1, m_2 = 7 \) are shown in Figs. 6.19 (A) and 6.19 (B), respectively. But, if the interfering MG beams have different orders parity and in phase configuration \((m_1 = 2, m_2 = 7 \) and \( m_1 = 13, m_2 = 14 \)) asymmetric intensity distributions are realized and depicted in Figs. 6.19 (C) and 6.19 (D), respectively.
Figure 6.19: Interference of two MG beams of different order. Transverse intensity distribution obtained by interfering MG beams with the same parity: (A) both even $m_1 = 0$ and $m_2 = 10$; (B) both odd $m_1 = 1$ and $m_2 = 7$, or different parity: (C) $m_1 = 2$ and $m_2 = 7$ (D) $m_1 = 13$ and $m_2 = 14$.

Figure 6.20: Interference of two MG beams of different order and phase configurations. Transverse intensity distribution of superimposing beams with the same parity: (A) $m_2 = 7$, $m_1 = 2$, $\pi$ out of phase; (B) $m_2 = 2$, $m_1 = 7$, $\pi/2$ out of phase; (C) $m_2 = 13$, $m_1 = 14$, $\pi$ out of phase; (D) $m_2 = 13$, $m_1 = 14$, $\pi/2$ out of phase.

In the previous cases, both interfering beams are in phase. In the case of the $\pi$ out of phase interference, mirror-symmetric structures are revealed (Fig. 6.20 (A), (C)). It is possible to observe symmetric structures by interfering MG beams of different orders only if they have phase differences of $\pi/2$ (Fig. 6.20 (B), (D)). The comparable structures could be produced by synthesizing mirror-symmetric structures.
In second approach two even MG beams with the same order \( m \), but oriented at 90° with respect to each other are superimposed, considering additionally the in-phase and \( \pi \) out of phase configurations and results are depicted in Figs. 6.21 and 6.22, respectively. First, two MG beams of even order parity \( m \) are superimposed and results for \( m = 2 \) and \( m = 8 \) are presented in Figs. 6.21 (A), (B) and Figs. 6.22 (A), (B) for in-phase and \( \pi \) out of phase configurations, respectively. Afterward, MG beams of odd order parity (\( m = 5 \) or \( m = 7 \)) are used and results are depicted in Figs. 6.21 (C), (D) for in-phase configurations and in Figs. 6.22 (C), (D) for \( \pi \) out of phase configurations.

![Figure 6.21: Transverse interference patterns of two MG beams of the same order \( m \), in-phase and oriented at 90° with respect to each other: even parity (A) \( m = 2 \); (B) \( m = 8 \); and odd parity (C) \( m = 5 \); (D) \( m = 7 \).](image)

![Figure 6.22: Transverse interference patterns of two MG beams of the same order \( m \) oriented at 90° with respect to each other in \( \pi \) out of phase configurations: even parity (A) \( m = 2 \); (B) \( m = 8 \); and odd parity (C) \( m = 5 \); (D) \( m = 7 \).](image)

All results are variable according to phase configurations. It is noticeable that for interference of MG beams with even order parity distinguish structures are observed for two different phase con-
figurations, while for MG beams with odd order parity the same intensity distributions are observed but mirror-symmetric to each other. This mirror symmetry of superimposed MG beams with odd orders $m$ is related to the intrinsic symmetry of the related Mathieu functions.

The next approach for the realization of 2D complex aperiodic structures is established on the superposition of MG beams at different mutual distances. The most opportunities for the realization of different patterns are provided by this approach. Figure 6.23 shows the interference of two even MG beams of the same order, $m = 6$ or $m = 7$ arrange along $x$-axis at various mutual distances $D$, 2D and 3D, where $D = 20 \mu m$.

![Numerics vs. Experiment](image)

**Figure 6.23:** interference of MG beams with same order at different mutual distances along $x$-axis: (A), (B) $m = 6$ and (C), (D) $m = 7$. First row: interference at mutual distance $D = 20 \mu m$, second: doubled distance 2D, and third: triple distance 3D.

In order to generate different complex aperiodic photonic structures via Mathieu beams, the previous approach of synthesizing multiple MG beams at different mutual distances is used. By using previous approaches, new field distributions are provided and they serve as a "unit cell" for more complex aperiodic beams. Such complex aperiodic beams could be used as writing light capable of being transferred to tailored refractive index modulations in photosensitive media i.e. photonic lattices. The unit cell would be multiplied in $x$ or $y$ direction which allows continuously increase the degree of aperiodicity.
In continuation new aperiodic photonic structures would be produced. First, pattern from Fig. 6.19 (B) is used as unit cell and multiply twice in x-direction at the distance of $D_x = 80 \mu m$. The single array is observed and presented in Fig. 6.24 (A). Afterward, the resulting array is multiplied along the y-direction at three different mutual distances $D_y = 80 \mu m$, $D_y = 88 \mu m$, and $D_y = 96 \mu m$. Various aperiodic lattice structures are observed as shown in Figs. 6.24 (B) - (D). Those examples exhibit areas where the initial unit cell is preserving its shape, but additionally novel unit cells appear in depends on the mutual distances between the multiplied arrays.

![Figure 6.24](image)

**Figure 6.24:** Realization of aperiodic photonic lattices by multiplying the structure from Fig. 6.19 (B) at various distances.

Next, the necklace structure from Fig. 6.22 (A) is investigated as unit cell. The unite cell is multiplied in x-direction at different distances. One example is presented in Fig. 6.25 (A), for $D_x = \text{...}$

![Figure 6.25](image)

**Figure 6.25:** Realization of aperiodic photonic lattices by multiplying the structure from Fig. 6.22 (A) at different distances.
144\mu m. Such observed structure is multiplied in y-direction to create a complex aperiodic structure by changing the mutual distances between them: \( D_y = [120, 144, 152] \mu m \) (Figs. 6.25 (C) - (D)). Initial unit cell and additional ones, which can be control by changing the distances between initial unit cell used for multiplying, are observed.

Figure 6.26: Various aperiodic photonic structures realized by multiplying the structure from Fig. 6.23 (A2) at different mutual distances.

Figure 6.26 presents novel 2D aperiodic photonic structures created via Mathieu beams. The unit cell depicted in Fig. 6.23 (A2) is multiplied along x-axis at different mutual distances: \( D_x = [152, 176, 192] \mu m \) and equivalent results are shown in Figs. 6.26 (A), (B), (C), respectively. By this, the initial structure shape is preserved but with slightly different interfering patterns between them. Then the structure from Fig. 6.26 (D) is multiplied along y-direction for \( D_y = 104 \mu m \), and a new 2D complex structure is observed.

Figure 6.27: Different aperiodic photonic structures created by multiplying the structure from Fig. 6.23 (C2) at different mutual distances.
Figure 6.27 presents novel 2D aperiodic photonic structures, with the structure from Figs. 6.23 (C2) as unit cell. Such structure is multiplied in x-axis for distance $D_x = 176\mu m$ and result is shown in Fig. 6.27 (A). Observed structure is multiplied along the y-direction at various mutual distances: $D_y = 72\mu m$, $D_y = 96\mu m$, and $D_y = 104\mu m$, and additional shapes of 2D aperiodic photonic structures are observe, as shown in Figs. 6.27 (B) - (D).

Such formed complex aperiodic structures would be use as writing light to create 2D photonic lattices in photosensitive SBN crystal and examine light propagation in them (next subsection). Therefore, their nondiffracting character have to be confirmed in SBN crystal. Two aperiodic structures, presented in Figs. 6.25 (D) and 6.27 (D), are investigated during linear propagation through the 20mm long crystal. Figures 6.28 (A) and (D) depict $xz$ cross-sections through the intensity volume at the orientation indicated with the white line in Figs. 6.28 (B) and (E), respectively. This $xz$ cross-section prove that the complex beams propagate invariant thought the SBN crystal. Experimental setup presented in Fig. 4.3 is used for optical induction of aperiodic photonic lattices by ordinary polarized writing beams. The illumination time is 35s with a moderate laser power of $P \approx 30\mu W$ and an external electric field of $E_{ext} = 2000$ V/cm. Due weak nonlinear self-action, lattice writing beams inscribe stationary photonic lattice in crystal. Numerically and experimentally intensity distribution at the back face of the crystal are represented in Figs. 6.28 (B) and (E).

Figure 6.28: Waveguiding in aperiodic photonic structures. As writing beam is used structure from: (A), (B): Fig. 6.25 (D); (D), (E): Fig. 6.27 (D). Intensity distribution of probe beam at the exit face of the crystal (C), (F).
In addition, the optically induced 2D aperiodic lattice were probe with an extraordinarily polarized plane wave to demonstrate waveguiding using experimental setup shown in Fig. 4.4. Figures 6.28 (C) and (F) demonstrate waveguiding of the initial plane wave in the 2D aperiodic lattices, manifested in a spatially modulated intensity distribution according to the underlying refractive index modulation. As expected, the intensity is preferentially guided in areas where the refractive index is increased and high intensity spots are formed.

In this research, an approaches for realization of numerous new aperiodic patterns by coherently superimposing Mathieu-Gauss beams with different orders, positions, and relative phases is presented. The various 2D aperiodic photonic Mathieu lattices are created by the optical induction technique in SBN crystal. This research is extend, with light propagation study in such aperiodic photonic lattices.

### 6.3.2 Light propagation in aperiodic photonic lattices created by synthesized Mathieu-Gauss beams

In this section, the effects of light propagation in the aperiodic photonic lattices created by synthesized MG beams in a SBN crystal are investigated experimentally and numerically. The influence of various input beam positions on light diffraction is investigated in linear and nonlinear regimes.

Figure 4.4 shows the experimental setup to fabricate and probe MG beam based photonic lattices. Photonic structure is probed with extraordinarily polarized Gaussian probe beams to feel stronger nonlinearity effect. Figs. 6.29 (A) and (B) present an aperiodic lattice and the characteristic lattice unit cell. The probe beam is launched into a single site inside aperiodic lattice unite cell and experiences lateral transport within the lattice as it propagates along its axis. Thus, the probe beam propagation resulting in a diffraction pattern in dependence of the local structure, which is in contrast to a simple periodic lattice where probe beam expansion is the same for each initial site excitation. In an aperiodic lattice different local environments exist, hence the transport behavior is expected to vary significantly from site to site. Three excitation positions with different local environment are chosen as depicted in Fig 6.29 (B).

![Figure 6.29: Transverse intensity distribution of periodic and aperiodic lattices. (A) Aperiodic lattice created via MG beams, (B) typical unit cell, where the yellow arrows indicate the probe beam excitation sites. (C) Periodic square lattice with period $d$ equal to the characteristic structure size $a = 2\pi/k_\perp$ of used MG beams, $d = a = 25 \mu$m.](image-url)
First, the linear propagation of light is investigated in such lattice. The lattice is fabricated with
the external electric field of \( E_{\text{ext}} = 2000 \, \text{V/cm} \) and laser power of \( P_0 = 50 \, \mu \text{W} \), which corresponds to
a simulated lattice intensity of \( I_{\text{latt}} = I_0 = 0.7 \). After the lattice writing beam and the external electric
field \( E_{\text{ext}} \) are switched off, the Gaussian probe beam with a FWHM of \( w_0 = 8 \, \mu \text{m} \) and low power of
a few \( I = 10 \, \mu \text{W} \) illuminates one site into the lattice to display linear propagation.

Figure 6.30 presents the intensity distributions of the probe beam after propagating through the
lattice inside 20mm long SBN crystal for three distinguish excitation sites marked by numbers 1, 2,
and 3 in Fig. 6.29 (B). The first row in Fig. 6.30 shows transverse intensity distributions for those
three probing, obtained at the back face of the 20mm long SBN crystal. The propagation through the
lattice differs in dependency on which local lattice site is excited. Therefore, the discrete diffraction
profiles at the lattice output differ from each other.

Further, numerous experiments and numerical calculations are performed to observe nonlinear
localization of the discrete diffraction pattern. Consequently, the intensity of the probe beam is incre-
mented to increase nonlinearity strength. The second row of Fig. 6.30 shows the transverse intensity
distribution of the probe beam when propagating through the commonly fabricated lattice with a beam
power of \( P_0 = 50 \, \mu \text{W} \) or \( I = I_0 = 0.7 \). Next, the strength of the nonlinearity is increased by doubling
the beam power \( 2P_0 \) in the experiment and simulation, and the results are presented in the third row
of Fig. 6.30.

For the input position 1 and sufficiently high beam powers a \textit{spatial soliton} in the aperiodic
lattice are observed (Figs. 6.29 (C1), (C2)), while the other input positions 2 & 3 do not support this
localized state (Figs. 6.29 (E), (F), (H), (I) for the same beam power. Robustness of such spatial
soliton is examined numerically by changing the intensities of the probe beams while keeping all

![Image of Figure 6.30](image-url)
other parameters fixed, and it was found that such solitons remain unchanged an up to 3 times higher beam power. The stability of spatial soliton is studied numerically in dependence of the propagation distance and stable output intensity distribution is obtained up to 10cm (not shown).

For the investigation of localization properties of the aperiodic Mathieu lattice in general, independent of the concrete excitation site, the light propagation in the lattice is numerically simulated for 100 different probe beam excitation positions and their expansion is averaged. The probe beam is moved with equidistant spacing across the unit cell depicted in Fig. 6.29 (B). Figure 6.31 presents results for the identical intensities as in Fig. 6.30. The gradual transition from suppressed discrete diffraction to nonlinear localization is noticed. While nonlinearity increases, the output averaged transverse intensity profiles in Fig. 6.31 (A), (B), (C) narrow. But, due to the diverse contributions that are averaged (on- and off-lattice sites are included), their profiles shown in (D) do not show the soliton shape as typically known from spatial bright solitons in the bulk.

![Figure 6.31: Averaged intensity distributions at the lattice output, for 100 different probe beam excitation sites, in (A) linear and (B), (C) nonlinear cases. (D1) and (D2) present averaged intensity profiles, taken along the horizontal and vertical transverse direction (indicated with the white lines in (A)), respectively. Parameters are as in Fig. 6.30.](image-url)
The effective beam width $\omega_{\text{eff}} = \text{PR}(z)^{-1/2}$, is calculated to characterize the amount of beam expansion, where

$$\text{PR}(z) = \frac{\int |A(x,y,z)|^4 dxdy}{(\int |A(x,y,z)|^2 dxdy)^2}$$ (6.3.3)

is the inverse participation ratio [9]. For aperiodic lattices, it is useful to calculate effective width over different incident beam positions and to present averaging effective width in order to remove the effects of the local environment. The averaged effective beam width is calculated along with the propagation distance. A statistical analysis of the effective beam width for the cases demonstrated in Fig. 6.31 is performed. Figure 6.32 shows these results. Hence, it is noticeable that beam expansion during propagation is more hindered while the input beam power increase.

The propagation of light in Mathieu aperiodic lattice is compared with the propagation of light in periodic square lattice. The square lattice is created with period $d$ equal to the characteristic structure size $a = 2\pi/k_\perp$ of MG beams used to create the aperiodic lattice, $d = a = 25 \mu m$ (Fig. 6.29 (C)) (angular spectrum of aperiodic lattice and square lattice lie on same ring). It is demonstrated that light is less localized in the periodic square lattice than in the aperiodic lattice created by synthesizing MG beams.

![Figure 6.32: Comparison between beam spreading in linear and two nonlinear regimes inside the aperiodic lattice and an appropriate periodic square lattice. Numerical simulation of averaged effective width (averaged over 100 excitation positions) along the propagation distance. $\omega_{\text{in}}$ is the initial effective width. Parameters are as in Fig. 6.30.](image)

Aperiodic lattices are located between periodic and disorder lattices therefore gradual randomization of aperiodic lattices to completely disordered is an opportunity for a new investigation as well as an examination of light transport in such lattices. In the past research quasi-periodic crystals are used for transition to the disorder patterns and light propagation and transport are investigated. Here are introduced new aperiodic lattices realized by synthesizing MG beams, and they can be used for investigation of how disorder influences the light propagation.
An approach for a gradual transition from aperiodic to disorder lattices is presented by relation \( \text{Lattice} = AL(1 - p) + pDP \), where \( AL \) denotes aperiodic lattice, \( p \) percent of randomization and \( DP \) is disorder pattern. For the proper randomization of the aperiodic lattice, the Fourier spectrum of aperiodic and disorder patterns have to be pointed on the circle with radius \( k = 2\pi/a \) determined by characteristic structure size \( a \) of Mathieu beam used for the realization of aperiodic lattices. One example of a transition from a periodic lattice created via the superposition of MG beams to a complete disorder structure is represented in Fig. (6.33).

Which phenomena would be observed during light propagation in Mathieu lattices with some percent of disorder, how disorder or different aperiodic Mathieu lattices influence of light transport are some open questions. Realization of periodic, aperiodic and disorder lattices with Fourier components on the same ring, are suitable for comparison of light propagation. This enables new directions for the usage of Mathieu aperiodic lattices and futures researches.

![Figure 6.33: The transition from aperiodic Mathieu lattices to disorder lattices.](image)

This section presents new aperiodic lattices realized by MG beams, as well as experimental and numerical investigation of linear and nonlinear propagation of the Gaussian probe beam in them. It is observed that Mathieu beams, due to their different intensity distribution shapes create versatility aperiodic structures. This research endorses the realization of aperiodic lattices with the versatility in aperiodic which provides considerable flexibility and richness in light propagation modeling. It is observed that the excitation of the probe beam in dependency of the input beam position in aperiodic Mathieu lattice creates different diffraction of light. By comparing light propagation in such lattice and appropriate periodic lattice light it is demonstrated that in aperiodic latticed light is more localized. In such aperiodic lattices, light localization, such as robust spatial soliton, is observed for nonlinearly propagating probe beams. Moreover, an approach for the transition from the Mathieu aperiodic lattices to disorder lattices is introduced.
Chapter 7

Conclusion

Photonic lattices offer great potential for controlling and manipulating light propagation, therefore, they become an important research area of modern optics during the last decades. Many researches from this field are applicable in the development of other areas, particularly in information and communication technologies. Moreover, when the photonic lattices are inscribed in some nonlinear material, a combination of photonic lattice properties and nonlinearity provides a unique opportunity to achieve ultimate control over linear and nonlinear light propagation. Optical induction in the photorefractive medium provides a successful experimental realization of photonic lattices. Due to the fast and easy reconfiguration of refractive index modulations with optical induction technique easy fabrication of different photonic lattices is provided. Through this thesis different optically induction photonic lattices are realized and fundamental studies of wave propagation in such deterministic aperiodic photonic lattices are examined.

Nondiffracting Mathieu beams facilitate the realization of various photonic lattices in photorefractive crystal via optical induction, even using single Mathieu beams, elliptical Mathieu beams, as well as the superposition of multiple Mathieu beams. By this plentiful family of nondiffracting beams, photonic lattices with the configurable shape are realized in this thesis. Single Mathieu beams with manageable ellipticity $q$, and characteristic structure size $a$ provides realization of both 1D and 2D photonic lattices. Photonic waveguides are realized along different paths like straight layer, circle, ellipse, or even hyperbola only by using Mathieu beams. Also, higher-order Mathieu beams make progress in the realization of the periodic lattice with a manageable period between waveguides.

The realization of discrete aperiodic lattices and propagation of light in them is an unfasten area for research. Mathieu beams, due to diversity of intensity distribution, are great candidates for the creation of various discrete aperiodic lattices. In these thesis, several approaches for the realization of aperiodic lattices using the interference of Mathieu beams in different relations one to another are involved. Thus, different aperiodic photonic lattices are presented light propagation is investigated. Due to different environments in such lattices control of light propagation is possible in both linear or nonlinear regimes. Conditions for light localization are examined. Strong light localization such as robust spatial soliton formation with respect to intensity changes and propagation length is demonstrated in the nonlinear regimes in such lattices. Aperiodic lattices created by synthesizing Mathieu beams hinders the beam expansion during propagation compared to periodic lattices. These results open new directions to exploit light propagating in a broad range of aperiodic photonic lattices and may have applications in capacity-enhanced optical information processing.

The study of the nonlinear propagation of single Mathieu beams of different orders provides nonlinearity strength range in which Mathieu beams preserve nondiffracting character during propagation. Hence, in these range realization of nonlinear Mathieu lattices are feasible. In such nonlinear
lattices, a narrow Gaussian beam propagates the same as in 1D or 2D periodic lattices creating well-known discrete diffraction intensity distribution during the propagation. The potential of using Mathieu beams for the realization of such nonlinear lattices is due to the opportunity to arrange waveguides in the straight array as same as in the hyperbolic array. Therefore, in such nonlinear waveguides 1D discrete diffraction along the straight and hyperbolic waveguide layer is demonstrated.

Research of nonlinear propagation of elliptical Mathieu beams reveals that for very low nonlinearities elliptical Mathieu beams remain nondiffracting beams. Opposite, for higher nonlinearities, the intensity distribution is modulated in the form of high-intensities filament, and the elliptical Mathieu beams are no longer nondiffracting. It is demonstrated that during propagation, these high-intensity modulations along inner ellipse are rotating in the direction determined by energy flow i.e. Poynting vector. The filamentation is investigated in dependence of nonlinearity strength, stricter size, and order of Mathieu beam. Due to this, it is demonstrated that the number of filaments and the velocity of rotation filaments can be increased by increasing these parameters. By elliptical Mathieu beams, dynamical rotation structures are achieved inside SBN crystal.

According to the previous study about elliptical Mathieu beams, 2D chiral waveguides with manageable properties, as well as the possibilities to change the number of chiral waveguides or their slope are demonstrated. It is established that the strength of nonlinearity, order, and characteristic structure size of used Mathieu beams increase the number of chiral waveguides, while the characteristic structure size strongly influences the slope of waveguides. This is an improvement for the realization of 2D chiral photonic lattices with manageable properties. In past realization, chiral lattices are experimentally realized demanding a long realization time because every waveguide is separably inscribed into the crystal using complex experimental setups. By using elliptical Mathieu beams, chiral lattices can be fast and easily realized in photosensitive media by optical induction techniques. Such lattices open new fields for research of light propagation.

Due to investigation in this thesis new phenomena, nonlinear discrete diffraction, or morphing discrete diffraction are demonstrated by using Mathieu beams in nonlinear SBN crystal. 1D nonlinear discrete diffraction is specified by nonlinear propagation of zero-order Mathieu beam inside photorefractive media. When the order of the Mathieu beam increases also the number of spots over straight or hyperbolic layers increases, allowing dimensional cross over from one- to two-dimensional structures. Several higher-order Mathieu beams were investigated during nonlinear propagation in photorefractive media, and consequently, 2D nonlinear discrete diffraction is demonstrated.

Control of light propagation in Mathieu lattices is examined, such as the propagation of elliptical vortex in appropriate Mathieu lattices. It is demonstrated that elliptical vortex propagated in Mathieu lattices creating the stable elliptical vortex necklaces during propagation, oscillating dipoles, dynamic instabilities. The stability of vortex necklaces is examined for different Mathieu beam orders and ellipticities, different sizes, and topological charge of the elliptical vortex, over long propagation distances, and strength of nonlinearity. It is revealed that higher beam order provides more pearls in elliptical vortex necklaces, as well as the size of the elliptical vortex, while the ellipticity changes the distance between the pearls, while higher orders topological charges harm the stability of elliptical vortex necklaces. Higher orders topological charges visible influence on the phase distributions, creating phase separation which increases while the ellipticity, topological charge, and propagation distance increase. For longer propagation distances elliptical necklaces are stable for propagation length of few crystal sizes, but for further propagation distances, the oscillating dipole states are realized. According to nonlinearity strength, elliptical necklaces are stable only for low nonlinearities. In contrast, for higher nonlinearities, elliptical necklaces are damaged. This research supports the future examination of the elliptical vortex in different suitable lattices realized with optical induction technique. Such research leads to the potential application of elliptical vortex beams in optical tweezers, quantum information processing, or optical manipulation and trapping.
In this thesis different photonic lattices are featured, accomplished with Mathieu nondiffracting beams as well as manipulation of light transport in different Mathieu lattices. In particular, different Mathieu aperiodic lattices created in this thesis can be extended to studies of other deterministic aperiodic photonic lattices optically induced by other than Mathieu beams. Further examination of various aperiodic lattices would contribute to a fundamental understanding of light transport characteristics in these artificially designed structures. Such examination opens and new applications of aperiodic lattices to control of light or future devices realizations.
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