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FATIGUE LIFE ASSESSMENT OF DAMAGED INTEGRAL SKIN –STRINGER PANELS

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Nomenclature

\( a \) Crack length
\( a_0 \) Initial crack length
\( E \) Young’s modulus
\( F \) Applied force
\( J \)-integral
\( K \) Stress intensity factor
\( K_i \) Mode-I stress intensity factor
\( K_{\text{eff}} \) Effective stress intensity factor
\( \nu \) Poisson’s ratio
\( C \) Constant (in Paris equation)
\( m \) Constant (in Paris equation)
\( da/dN \) Fatigue crack growth rate
\( \Delta K \) Stress intensity factors range
\( R \) Stress ratio
\( N \) number of cycles
Abbreviations

BOAC British overseas aircraft company
FAR Federal Aviation Regulations
DSG design service goal
NTSB National Transportation Safety Board
DOC direct operating cost
SIF(s) stress intensity factor(s)
XFEM extended finite element method.
LEFM linear elastic fracture mechanics
CTOD crack tip opening displacement
FEM Finite Element Method
BEM Boundary Elements Method
BM base material
FSW Friction Stir Welding
LBW laser beam welding
SN stress level versus number of cycles (curve)
VCCT virtual crack closure technique
GDC generalized form displacement correlation method
MCCI modified crack closure integral
PUM Partition of unity method
DOF degrees of freedom
PUFEM partition of unity finite element method
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Introduction

1.2 Background

Typical riveted skin-stringers structures have been introduced in aircraft fuselage assemblies since the 1940’s, and then widely used in many parts of the aircraft (as shown in Figure 1.1). It seems that it is difficult to get significant improvement in this technology because of the advancement made during the last century. Integral skin-stringer structures, which make skin and stringers as a continuum, are suitable to change the situation, even though they are poor at damage tolerance performance. Compared with the conventional riveted structures, integral skin-stringer structures have many advantages, such as lower weight and lower cost to manufacture. It is worthy of note that, fewer components mean they are easy to inspect and no holes in riveted joints improve fatigue crack initiation life.

Figure 1.1 Locations of stringer panels in the aircraft
NASA began Integral Airframe Structures (IAS) Program to develop integral metallic structures in 1966 [1.1]. The purpose of the program was to design and test structures, which were lower in price than the current structures and improvement in structural weight and performance. The IAS program obtained satisfactory results with the improvement and the application of integrally stiffened fuselage structure. The configuration of integral aircraft fuselage structure and conventional fuselage structure are compared in Figure 1-2.

![Integral Aircraft Structure and Conventional Structure](image)

**Figure 1.2 Integral aircraft structure and conventional structure [1]**

In recent years, the technology of design, analysis integral structures have become one of the key technologies for the widespread use of the integral metallic skin-stringer structures in the aerospace field. Two different methods are used in order to optimize the damage tolerance performance of the integral skin-stringer panels. The first one is to apply new alloy materials with lower crack propagation rate and higher fracture toughness. Another one is to design or optimize new structure conformation. In order to achieve the latter objective, many researchers have been done research to develop efficient and reliable methods to improve the damage tolerance performance of integral skin-stringer panels [1.2].
1.2 Laser Beam Welding Joining Technology.

Riveting has been the state of the art joining technology in the aeronautical industry for decades and it has demonstrated its value and reliability. But the necessary overlap joint demands large amounts of material and its production chain is also time consuming. New processes such as LBW and Friction Stir Welding (FSW) present new solutions to overall weight savings and process time reduction. These processes are continuously under improvement and their application is still to be broadened. It has been observed in larger metallic structures of models such as in the A318, A340 and A380 that LBW has advantages compared to conventional riveted fuselages. Regarding the production of structures, LBW can be up to 20 times faster than riveting. LBW is characterized by high energy concentration with high welding speed, narrow Heat Affected Zones (HAZ), deep penetration effect and low remaining component distortion after welding [1.3]. Another advantage is that the process only requires one-sided access. Lower fuselage panels made of AA6xxx series (Al-Mg-Si-Cu) and processed with LBW as an efficient joining technology are already established in the market, figure 1.3. In fact, LBW has been applied with AA6013 and AA6056 as part of the skin and AA6110 or AA6056 for the stringer. These AA’s provide higher buckling strength and lower weight compared with riveted design [4]. High potential has also been found on the AA5xxx series for welded structures due to the ease to weld it and good properties after welding [1.4].

![Figure 1.3: LBW panels in the A380 [Airbus source.]](image)
The AA2xxx series, which is a copper alloy type, is highly used for aircraft sheet construction, as is the case of AA2024 fuselages but this specific AA is not good for welding. Now it is possible to use laser-weldable AA’s of the 2xxx series such as AA2198 [1.4]. Nevertheless, there are some limitations in the use of LBW. For the case of aluminum, when it is subjected to heat, it suffers large deformations which are due to its high thermal expansion coefficient and these may induce panel distortions. Low energy input is, therefore, desirable in order to reach the given production tolerances. Another limitation of welding aluminum is that it can not be in contact with air. Helium or argon are used as shielding inert gases around the area where the welding process takes place. Another problem of welding materials with lithium is the large pore formation in laser weld seams [1.3]. As a consequence, the strength in the HAZ is reduced, sometimes by as much as 50%, due to the thermal treatment the material receives during welding. The introduction of the LBW method requires a transfer from riveted differential build up structures to laser welded integral structures, as seen in figure 1.4, and the introduction of high strength materials. One of the current challenges for LBW is to provide appropriate, crack free joints with low porosity, resulting in high mechanical performance of the welded joint [1.5].

Figure 1.4: Difference between crack in a differential riveted and integral welded structures [1.6].

Despite manufacturing precautions, cracks may appear in integral skin-stringer structure and reduce its stiffness and the load-carrying capacity. The safe operation of pressurized fuselage structure is ensured through the operation of damage tolerant design and evaluation, which
intended to certify that fatigue, corrosion, or other mechanisms should cause crack to grow within
the operational life, so that the remaining structure can withstand reasonable loads without failure
or excessive structural deformation until the damage is detected. Two types of damage most
frequently associated with the structural integrity of the fuselage are longitudinal cracks under
hoop stresses (induced by cabin pressurization) and circumferential cracks under stresses from
vertical bending of the fuselage [1.7]. A critical element of damage tolerant design in pressurized
fuselage is the ability to predict the growth rate of fatigue cracks under known applied loading.

The crack growth stage is studied by using stress intensity factor (SIF). SIF is fundamental quantity
that governs the stress field near the crack tip. It depends on the geometrical configuration, crack
size and the loading conditions of the body. There are many methods used in the numerical fracture
mechanics for SIFs calculations. The crack opening displacement (COD) method, as well as the
force method, was popular in early applications of finite element method (FEM) to fracture
analysis [7]. The virtual crack extension (VCE) methods, proposed by Parks [1.8] and Hellen [1.9],
led to increased accuracy of SIF results. The VCE method requires only one complete analysis of
a given structure to calculate SIF. Both the COD and VCE methods can be used to calculate SIF
for all three fracture modes. However, additional complex numerical procedures have to be applied
to get results.

FEM has been used for decades for calculating SIFs, but it has some restrictions in crack
propagation simulations mainly because the finite element mesh needs to be updated after each
propagation step in order to track the crack path. Extended finite element method (XFEM)
suppresses the need to mesh and remesh the crack surfaces and is used for modeling different
discontinuities in 1D, 2D and 3D domains. XFEM allows for discontinuities to be represented
independently of the FE mesh by exploiting the Partition of unity finite element method (PUFEM)
[1.10]. In this method additional functions (commonly referred to as enrichment functions) can be
added to the displacement approximation as long as the partition of unity is satisfied. The XFEM
uses these enrichment functions as a tool to represent a non-smooth behavior of field variables.

There are many enrichment functions for a variety of problems in areas including cracks,
dislocations, grain boundaries and phase interfaces. Recently, XFEM and its coupling with level
set method were intensively studied. The level set method allows for treatment of internal
boundaries and interfaces without any explicit treatment of the interface geometry.
Due to the relatively short history of the XFEM, commercial codes which have implemented the method are not prevalent. There are however, many attempts to incorporate the modeling of discontinuities independent of the FE mesh by either a plug-in or native support. Cenaero [1.11] has developed a crack growth prediction add-in Morfeo/Crack for Abaqus which relies on the implementation of the XFEM method available in Abaqus software. Problems involving static cracks in structures, evolving cracks, cracks emanating from voids etc., were numerically studied and the results were compared against the analytical and experimental results to demonstrate the robustness of the XFEM and precision of Morfeo/Crack for Abaqus [1.12].

The cracks are represented with the help of two signed distance functions that are discretized on the same mesh as the displacement field with first-order shape functions. Method for representing the cracks in this application is exactly the same as described in [1.13]. After each step of the propagation simulation, the SIFs are computed from the numerical solution at several points along the crack fronts. Interaction integrals are used to extract the mixed-mode SIFs with the help of auxiliary fields. After that Paris-Erdogan fatigue crack growth model [1.14] based on the range of the stress intensity factor can be used for evaluation of the number of cycles that will grow crack to the critical length. This model is chosen because of its simplicity; however, it has been used for decades as a basic framework in fracture mechanics.

1.3 Problem definition and adopted approaches.

The aerospace industry uses riveting technology to join metallic materials for designing airframe structures. Riveting technology causes damage tolerance behavior of riveted/fastened aircraft structures. Fatigue crack initiation and growth, as well as fracture resistance and corrosion issues associated with riveted structures are well understood.

Recent developments in Al-alloys and welding technologies have led to the use of advanced welding technologies to reduce weight and fabrication costs. Laser beam welding (LBW) has been successfully applied for manufacturing of “skin-stringer” panels for various civilian aircrafts in Europe. The fabrication of stiffened panels to join skin-stringer (T-joint) parts for integral airframe structures were made of 2xxx and/or 6xxx series wieldable Al-alloys. This technological development will eventually lead to use of LBW process in joining of further components such as
“skin-clip”, “clip-frame” or “frame-skin”. This application achieves further weight and cost savings.

The basis for this thesis is the report presented on “European Workshop on Short Distance welding Concepts for airframes - WEL-AIR” on June 2007 that is created on the basis of damage tolerance analysis of 4-stringer flat panels that are jointly made by the Airbus division in Bremen and GKSS Research Center Geesthacht (Hamburg) – Germany. By courtesy of project participants, the results of fatigue test of laser beam welded short distance clip welds using 4-stringer flat panels, were available for inspection and they were used as reference for verification of fatigue life values obtained by numerical simulations using XFEM.

1.4 Thesis Outline

This research is divided into 5 chapters, which look at achieving the above objectives; the details of each chapter are given below:

- **Chapter 1**: Introducing the importance of the development of welding technology.
- **Chapter 2**: presents a critical review of the issues affecting fatigue design approaches.
- **Chapter 3**: In this chapter introduces briefly the analysis methods for SIF (stress intensity factor) Calculation and crack growth life prediction for integral stiffened panels.
- **Chapter 4**: In this chapter, Numerical simulations of crack growth in damage integral skin stringer panel using XFEM.
- **Chapter 5**: Experimental Validation of Numerical Results (XFEM).
- **Chapter 6**:
- **Chapter 7**: This chapter concludes the results and discussion.
CHAPTER 2: LITERATURE SURVEY.

2.1. HISTORICAL ACCIDENTS RELATED TO FUSELAGE FATIGUE.

This Chapter presents a critical review of the issues affecting fatigue design approaches. The applications of damage tolerant methodology for plates with stringer reinforcement have been also presented. The design of aircraft including the development, and testing programs are mentioned. The safety is of the foremost concern to the aircraft design. The investigations normally lead to new research and development, improved design, and modified regulations.

2.1.1 Damage Tolerance Design Philosophy.

Historically, there are three different approaches to ensure the safety of an aircraft structure: Safe-life, fail-safe and damage tolerance [2.1]. Safe-life, introduced in the 1930’s, takes a philosophy of safety by retirement. The life of a structure is the number of flights, landings, or flight hours during which there is a low probability that the strength will degrade below its design strength. It assumes that, throughout its entire life, all identified loads are low enough and the strength high enough to sustain them. In the fail-safe philosophy, introduced in the 1950’s, a structure is capable of sustaining a certain amount of damage without catastrophic failure of the entire structure. To ensure this, the design approach considers additional load path elements in case one of the elements fails. The specification of necessary inspection intervals is based on the service experience and does not consider the initiation and growth of cracks. Finally, introduced in 1978, the damage tolerance philosophy assumes that the structure contains an initial defect that will grow under service usage. It requires fracture mechanics based engineering evaluation of crack growth and residual strength characteristics to establish the inspection intervals. The main objective is to detect and monitor cracks in the structural elements before they propagate to failure. In the Federal Aviation Regulations (FAR), part 25, section 571, it is stated that” an evaluation of the strength, detail design and fabrication must show that the catastrophic failure due to fatigue, corrosion, or accidental damage, will be avoided throughout the operational life of the air plane” [2.2]. For all primary structures with the exception of the landing gear the damage tolerance design and
evaluation is required [2.3]. A crack extending into two frame bays with the central frame also cut is generally assumed. The structure is considered to comply with the regulations if, under the specified conditions, it arrests the skin within two frame bays.

On January 10, 1954, a British Overseas Aircraft Company (BOAC) de Havilland Comet I aircraft, on its way to London from Rome, suffered a midair disintegration at about 30,000 feet and crashed into the Mediterranean Sea off the island of Elba. At the time of the accident, the aircraft, registration number G-ALYP (known as Yoke Peter) had flown 3,680 hours and had experienced 1,286 pressurized flights [2.4]. Comet I aircraft was the first high-altitude transport jet aircraft ever flown, which enabled the aircraft to achieve a much higher altitude, therefore extending its range and increasing its efficiency. This aircraft was capable of maintaining a cabin pressure differential of almost twice that of any other aircraft in service at that time [2.4]. The Comet I fleet was grounded for modifications after the GALYP incident. Two weeks after reinstatement, on April 8, 1954, a second Comet (Yoke Yoke), on its way to Cairo from Rome, crashed near Naples. Yoke had accumulated 2,703 flight hours and 903 flight cycles. The Comet accidents were later found to be related to fatigue cracks caused by the high stresses at corners of the automatic direction-finding window. The fuselage cabin for this airplane was substantiated by static pressurization. These accidents raised concerns on how fatigue is addressed in the regulations for jet transport aircraft, and resulted in inclusion of fatigue requirements in the regulations.

2.1.2. Aloha Airlines Accident

On 28 April 1988, Aloha airlines Flight 243, a Boeing 737-200, suffered a midflight explosive decompression while undertaking a regularly scheduled passenger flight from Hilo to Honolulu, Hawaii. The explosion, which occurred at 24,000 feet above the Pacific Ocean, caused a disintegration of a 17ft section of the crown of the fuselage.

There were 95 people on board this flight: 89 passengers, two-flight crew, three flight attendants, and an FAA air traffic controller in the jump seat. Remarkably, the only fatality was the senior flight attendant who was sucked out of the aircraft during the explosive decompression. A photo of the aircraft after the accident is shown in Figure 2.1

The aircraft had been placed in service in 1969, and had accumulated 35,496 flight hours and 89,680 flight cycles. The design service goal (DSG) of that aircraft was 75,000 cycles. The 19 year old aircraft had averaged about 13 flights a day during its time of service, and had the second
highest flight cycles in the entire B737 fleet. In the accident report [2.5], the National Transportation Safety Board (NTSB) concluded, among others, that the fuselage failure initiated at the lap joint along stringer S-10L as a result of multiple site fatigue cracking along the upper rivet row of the lap joint combined with disbonding of the lap joint. Because of the disbond, there were high stress concentrations at the knife-edges of the countersunk rivet holes, resulting in fatigue crack initiation. The NTSB concluded that the long term effects of disbonding, the associated corrosion, and fatigue cracking in lap joints was not considered in the 150,000-cycle test during certification. Response to the Aloha Airlines accident included research initiatives, industry activities, and government regulations. [2.6]

![Figure 2.1. Catastrophic accident of Aloha Airlines B737.](image)

**2.1.3 Industrial revolution-1960**

Fatigue is a process of local strength reduction. The phenomenon is often referred to as a process of damage accumulation in a material undergoing fluctuating loading. This process occurs in engineering materials such as metallic alloys, polymers and composites. To describe the
mechanical fatigue process as a result of a repeated load working on a structure, different parameters are used, like cyclic load, stress intensity and crack growth rate. The maximum load is $P_{\text{max}}$, the minimum $P_{\text{min}}$ [kN] and the ratio between the minimum and maximum load ($P_{\text{min}}/P_{\text{max}}$), that is often used as a measure of the mean stress, is called the load ratio $R$. Crack growth rate $\text{d}a/\text{d}N$ is the crack increment $\text{d}a$ per loading cycle increment $\text{d}N$. The stress intensity factor $K$ [MPa$\sqrt{\text{m}}$], working on the crack tip is calculated from the applied load $P$ and actual crack length and direction in a construction. The maximum stress intensity is $K_{\text{max}}$, the minimum $K_{\text{min}}$ and the difference between both is $\Delta K$, see figure 2.2. Fluctuating loads can lead to fluctuating local high stresses and microscopic small cracks may appear. Once a crack exists in a structure, it will tend to grow under cyclic loading. Even if the maximum of the cyclic load on a construction is below the elastic limit of the material, fatigue may lead to failure. Fatigue is a progressive process the damage develops slowly in the early stages and near the end of a structure’s life, and it accelerates very quickly towards failure.

![Figure 2.2. Mechanical parameters to describe the fatigue loading system.](image)

However, details of the fatigue process may differ between materials. The fatigue process can be defined generally as [2.7] “The process of the cycle-by-cycle accumulation of local damage in a material undergoing fluctuating stresses and strains.”

- description of the phenomenon.
Fatigue of metals in structures has been studied since the beginning of the 19th century. Railroads, bridges, steam engines: a whole gamut of new structures and machines were developed, which were made of steel in the times of the industrial revolution. Many of them were exposed to cyclic stresses during service life and many of them failed, the origin of failure was unknown, until Albert [2.8] made the first report about failure caused by fatigue, in 1829. He observed failure of iron mine-hoist chains, caused by repeated small loads. Ten years later, in 1839, Poncelet, a professor of mechanics at the école d'application, Metz, introduced the term fatigue in his lectures. Rankine [2.9] recognized the importance of stress concentration in 1843. He noted that fracture occurs near sharp corners. However, until then the phenomenon was described qualitatively only.

- **Systematic experiments and microscopic observations.**

Wöhler made a major step in 1860. Wöhler, a railroad engineer, started performing systematic experimental research on railroad axles. He observed that steel would rupture at stress below the elastic limit if a cyclic stress were applied. However, there was a critical value of cyclic stress, the fatigue limit, below which failure would not occur. He found a way to visualize “time to failure” for specific materials. In this S-N-curve approach the stress amplitude, $\sigma_a$ is plotted as function of the number of cycles to failure, see figure 2.3 [2.10].

![S-N curves for low-carbon steel (fatigue limit) and AA 2014 (no fatigue limit).](image)

**Figure 2.3.** S-N curves for low-carbon steel (fatigue limit) and AA 2014 (no fatigue limit).
A logarithmic scale is used for the horizontal axis, while the stress is plotted using either a linear or logarithmic scale.

**Fatigue limit:** the stress below which a material can be stressed cyclically for an infinite number of times without failure.

**Fatigue strength:** the stress at which failure occurs for a given number of cycles.

The first crack surface investigations were made by Ewing [2.11] in 1903. He showed the nature of fatigue cracks, using a microscope, see figure 2.4.

![Crack surface showed by Ewing & Humfrey, 1903](image)

**Figure 2.4. Crack surface showed by Ewing & Humfrey, 1903**

- **Explanations and predictions.**
Around 1920 Griffith investigated the discrepancy between the theoretical strength of a material, and the true value, sometimes 1000 times less than the predicted value. He discovered that many microscopic cracks and/or other imperfections exist in every material. He assumed that these small cracks lowered the overall strength. Because of the applied load, high stress concentrations are expected near these small cracks, which magnify the stresses at the crack tip. These cracks will grow more quickly, thus causing the material to fail long before it ever reaches its theoretical strength. Any voids, corners, or hollow areas in the internal area of the material also result in stress concentrations. Mostly fracture will begin in one of these areas, simply because of this phenomenon [2.12].

2.1.4 after 1960: Paris and Elber

An important push to understand the fatigue process was made by Paris and Elber. In 1961, Paris found a more or less linear correlation on double logarithmic scales between crack growth rate da/dN and cyclic stress intensity factor ΔK for some part of the fatigue curve, see figure 2.5 [2.13].

This well-known Paris’ law reads:
\[
\frac{da}{dN} = C\Delta K^m
\] (2.1)

Where ΔK = Kmax - Kmin and C and m are experimentally determined scaling constants.
Figure 2.5. Paris’ Law: linear correlation between crack growth rate $da/dN$ and stress intensity factor $K$ on log-log scale.

Paris’ law is generally accepted for a wide range of different materials; however, the physical meaning is limited. The major issue at that time was how to explain stress ratio effects. In 1970, Elber published a famous article titled “Fatigue Crack Closure under Cyclic Tension” [2.14]. In this article, he assumed crack closure to be the cause of stress ratio-effects. By crack closure, he meant contact of the crack surfaces, at a load above the minimum load. Elber assumed that, when crack closure occurs, the effective cyclic stress intensity range $\Delta K_{\text{eff}}$ that works on the crack tip, is lower than the expected or applied $\Delta K$-range, see figure 2.6. The crack growth rate is no longer a result of the whole $\Delta K$ magnitude, but only of a part of it.
2.2 Fracture Mechanics

When performing a proof of strength, flawless components can generally be assumed. Under certain circumstances, potential discontinuities are taken into account with increased safety factors. Yet the existence of defects and cracks can fundamentally alter the strength behavior of components and structures. For example, technical products sometimes fail well below the static strength level or fatigue strength of the material. Technical fracture mechanics an interdisciplinary subject linking engineering mechanics and materials engineering, assumes the existence of cracks in components and structures as a matter of principle. Cracks can possess small dimensions in the micrometer range, but they can also be relatively large in size, e.g. in the range of a millimeter or even a meter. Typical crack types that often arise in components and structures.

The basis for fracture-mechanical concepts and methods is the investigation of circumstances in the immediate vicinity of the crack tip. By looking at local stresses in the area of the crack, stress and displacement fields appearing there, stress intensity factors and the fracture-mechanical material parameters that are relevant for cracks, concepts and methods are developed that make it possible to assess and predict stable and unstable crack growth. These basic circumstances and relationships will be described in the following.
2.2.1 Cracks and Crack Modes

Cracks are local separations of the material of a structure. These material separations disrupt the force flow in the component considerably. The force flow is sharply redirected, and a local singular stress field appears in the area of the crack tip or the crack front. Figure 2.7 b, c show the disturbance of the flow of force by cracks in comparison with a component without cracks, Fig. 2.7a.

![Figure 2.7 Disturbance of the force flow path through cracks.](image)

Force flow is defined as the transmission of forces or stresses through a component. Force flow lines can also be understood as stress level lines. Where force flow lines are sharply redirected and lie close to each other, high local stresses occur. A tensile-loaded plate without defects or cracks has a completely undisturbed force flow, Fig. 2.7 a. In a component with an edge crack, the force flow lines are sharply diverted and compressed due to the crack, Fig. 2.7b. A stress concentration arises at the crack tip of—purely theoretically—infinitely high stresses. Figure 2.7 c shows the force flow path of a component with an inclined internal crack. Force transmission through the component has again been disturbed considerably.

However, in contrast to the force flow in Fig. 2.7 b, the force flow path is now asymmetrical with respect to the crack. Obviously, the crack (or in the vicinity of the crack) is being loaded differently.
in Fig. 2.7b than in Fig. 2.7c. Because of the simplicity of crack geometry—a crack is regarded as a mathematical section in fracture mechanics—only three basic crack loading types (loading modes) exist for all cracks appearing in components and structures see Fig. 2.8

2.2.2 Mode I
Mode I, Fig. 2.8 a, encompasses all normal stresses that cause the crack to open, i.e., the crack edges to be removed symmetrically with respect to the crack plane. A pure state of mode I stress thus always exists when there is a symmetrical force Flow path with respect to the crack plane (see Fig. 2.7b). This is the case, for example, in tensile-loaded and bending-loaded components when the crack grows perpendicular to the normal stress. Since extended fatigue crack growth occurs under the influence of normal stress, fatigue cracks whose loading direction does not change in the cracking process are generally in a state of mode I loading.

![Diagram of Mode I, Mode II, and Mode III](image)

Figure. 2.8 The three basic crack loading types in fracture mechanics.

2.2.3 Mode II
Mode II, Fig. 2.8 b, is associated with all shear stresses that engender opposed sliding of the crack surfaces in the direction of the crack. This is the case, for example, in components that are, either globally or locally, under the influence of plane shear loading.

2.2.4 Mode III
Mode III, Fig. 2.8c, corresponds to the non-plane shear stress state, which causes the crack surfaces to move against each other at a right angle to the crack direction, i.e., in the direction of the crack front. Mode III loading can be encountered, for example, in torsional loaded shafts when the crack is found in a plane that is perpendicular to the shaft axis.

### 2.2.5 Mixed Mode

The basic crack loading types described above can also appear in combination called mixed-mode loading. It is a plane mixed-mode situation when, for example, mode I and mode II are superimposed. This is the case e.g. in a component with an inclined internal crack (see Fig. 2.7 c). Mixed-mode loading can be recognized, among other ways, by its asymmetrical force flow distribution with respect to the crack, Fig. 2.7 c. If all three crack stress types are superimposed, it is referred to as a general or spatial mixed mode state. This is associated, for example, with surface cracks, internal cracks or edge cracks lying at an angle to the loading direction within the component or on the component surface, or cracks in multiaxially loaded components. [2.15]

### 2.3 Fundamentals of Fatigue Crack Propagation

Fatigue crack propagation, referred to as stage II in Figure 2.12, represents a large portion of the fatigue life of many materials and engineering structures. Accurate prediction of the fatigue crack propagation stage is of utmost importance for determining the fatigue life. The main objective of the fatigue crack propagation may be presented in this form: "Determine the number of the cycles $N_c$ required for a crack to grow from a certain initial crack size $a_0$ to the maximum permissible crack size $a_c$, and the form of this increase $a = a(N)$, where the crack length a corresponds to $N$ loading cycles."
Fatigue crack propagation data are obtained from pre-cracked specimens subjected to fluctuating loads, and the change in crack length is recorded as a function of loading cycles. The crack length is plotted against the number of the loading cycles for different load amplitudes. The stress intensity factor is used as a correlation parameter in analyzing the fatigue crack propagation results. The experimental results are usually plotted in a log (ΔK) versus log (da/dN) diagram, where ΔK is the range of the stress intensity factor and da/dN is the crack propagation rate. The load is usually sinusoidal with constant amplitude and frequency. Two of the four parameters $K_{\text{max}}$, $K_{\text{min}}$, $\Delta K = K_{\text{max}} - K_{\text{min}}$ or $R = K_{\text{min}} / K_{\text{max}}$ are needed to define the stress intensity factor variation during a loading cycle.

A typical plot of the characteristic sigmoidal of a log(ΔK)-log(da/dN) fatigue crack growth rate curve is shown in Figure 2.9. Three regions can be distinguished. In region I, da/dN diminishes rapidly to a very small level, and for some materials there is a threshold value of the stress intensity factor range $\Delta K_{\text{th}}$ meaning that for $\Delta K < \Delta K_{\text{th}}$ no crack propagation takes place. In region II there is a linear log(ΔK) - log(da/dN) relation. Finally, in region III the crack growth rate curve rises and the critical stress intensity factor $K_c$, leading to catastrophic failure. Experimental results indicate that the fatigue crack growth rate curve depends on the ratio $R$, and is shifted toward higher da/dN values as $R$ increases.

Figure (2.9) typical fatigue crack growth behavior in metals.
Cyclic stresses resulting from constant or variable amplitude loading can be described by two of a number of alternative parameters. Constant amplitude cyclic stresses are defined by three parameters, namely a mean stress, $\sigma_m$, a stress amplitude, $\sigma_a$, and a frequency $\omega$, $\nu$. The frequency is not needed to describe the magnitude of the stresses. Only two parameters are sufficient to describe the stresses in a constant amplitude loading cycle. It is possible to use other parameters; for example, minimum stress, $\sigma_{\min}$, and the maximum stress, $\sigma_{\max}$, to describe the stresses completely. The stress range, $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$, can also be used in combination with any of the others, except, $\sigma_a$. In addition, another parameter is often convenient. This is the so-called stress ratio $R$, defined as $R = \sigma_{\min} / \sigma_{\max}$.

One of the above parameters can be replaced by the load ratio $R$ to define the cyclic load. Any of the following combinations fully defines the stresses in a constant amplitude loading: $\Delta\sigma$ and $R$, $\sigma_{\min}$ and $R$, $\sigma_{\max}$ and $R$, $\sigma_a$ and $R$, and $\sigma_m$. and $R$. The case of $R=0$ defines the condition in which the stress always rises from, and returns to 0. When $R=-1$, the stress cycles around zero as a mean, which is called fully reversed loading.

In order to study the parameters, which affect the fatigue crack growth a through thickness crack is considered in a wide plate subjected to remote stressing that varies cyclically between constant minimum and maximum values. The stress range is defined as $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$. The fatigue crack propagation rate is defined as the crack extension, $\Delta\alpha$, during a small number of cycles, $\Delta N$, the propagation rate is $\Delta\alpha / \Delta N$, which in the limit can be written as the differential $\frac{da}{dn}$. It has been found experimentally that provided the stress ratio $R = \sigma_{\min} / \sigma_{\max}$, is the same then $\Delta K$ correlates fatigue crack growth rates in specimens with different stress ranges and crack lengths and also correlates crack growth rates in specimens of different geometry.

This correlation is presented in Figure 2.10. The data obtained with a high stress range, $\Delta\sigma_{\text{high}}$, commence at relatively high values of $da/dN$ and $\Delta K$. The data for a low stress range, $\Delta\sigma_{\text{low}}$, commence at lower values of $da/dN$ and $\Delta K$, but reach the same high values as in high stress range case.
Figure (2.10) Correlation of fatigue crack propagation data by $\Delta K$ when the stress ratio, $R$, is the same.

In addition, the stress ratio $R$ can have a significant influence on the crack growth behavior. In other words, besides the stress intensity factor range, $\Delta K$, there is an influence of the relative values of $K_{\text{max}}$ and $K_{\text{min}}$, since $R = \sigma_{\text{min}} / \sigma_{\text{max}} = K_{\text{max}} / K_{\text{min}}$. This is presented in Figure 2.11, which shows that crack growth rates at the same stress intensity range $\Delta K$ values are generally higher when the load ratio $R$ is increasing. It is important at this point to note that the effect of the load ratio $R$ has proved to be from the bibliography strongly material dependent. [2.16]
A number of different quantitative continuum mechanics models of fatigue crack propagation have been proposed in the literature. All these models lead to relations based mainly on experimental data correlations. They relate da/dN to such variables as the external load, the crack length, the geometry and the material properties. One of the most widely used fatigue crack propagation laws is that proposed by Paris and Erdogan and is usually referred in the literature as the "Paris law". It has the form:

\[
\frac{da}{dN} = C (\Delta K)^m
\]  

(2.1)

Where \(\Delta K = K_{\max} - K_{\min}\), with \(K_{\max}\) and \(K_{\min}\) referring to the maximum and minimum values of the stress intensity factor in the load cycle. The constant \(C\) and \(m\) are determined empirically from a log(\(\Delta K\)) - log(da/dN) plot. The value of \(m\) is usually taken equal to 4 for aluminum alloys, resulting in the so-called "4th power law" while the coefficient \(C\) is assumed to be a material constant. Paris Law equation represents a linear relationship between log (\(\Delta K\)) and log (da/dN) and is used to describe the fatigue crack propagation. Experimental data are well predicted using Paris Law equation for specific geometrical configurations and loading conditions.
The effect of mean stress, loading and specimen geometry is included in the constant C. ("Paris law") has been widely used to predict the fatigue crack propagation life of engineering components.

The crack growth mechanism shows that a fatigue crack grows by a small amount in every load cycle. Growth is the geometrical consequence of slip and crack tip blunting. Re-sharpening of the crack tip upon unloading sets the stage for growth in the next cycle. It can be concluded from this mechanism that the crack growth per cycle, $\Delta a$, will be larger if the maximum stress in the cycle is higher (more opening) and if the minimum stress is lower (more re-sharpening). The local stresses at the crack tip can be described in terms of the stress intensity factor $K$, where $K = \beta \sigma \sqrt{\pi a}$, if $\sigma$ is the nominal applied stress. In a cycle, the applied stress varies from $\sigma_{\text{min}}$ to $\sigma_{\text{max}}$ over range $\Delta \sigma$. Therefore, the local stresses vary in accordance with:

$$K_{\text{min}} = \beta \sigma_{\text{min}} \sqrt{\pi a}$$
$$K_{\text{max}} = \beta \sigma_{\text{max}} \sqrt{\pi a}$$
$$\Delta K = \beta \Delta \sigma \sqrt{\pi a}$$

(2.2)

An amount of crack growth is defined as $\Delta a$ in one cycle, which is expressed in m/cycle. If growth were measured over e.g. $\Delta N = 10000$ cycles, the average growth per cycle would be $\Delta a / \Delta N$, which is the rate of crack propagation. In the limit where $N \rightarrow 1$, this rate can be expressed as the differential $da/dN$ When a structural component is subjected to fatigue loading, a dominant crack reaches a critical size under the peak load during the last cycle leading to catastrophic failure. The basic objective of the fatigue crack propagation analysis is the determination of the crack size, $a$, as a function of the number of the cycles, $N$. Thus, the fatigue crack propagation life $N_p$ is obtained.

When the type of the applied load and the expression of the stress intensity factor are known, application of one of the foregoing fatigue laws enables a realistic calculation of the fatigue crack propagation life of the component. As an example, consider a plane fatigue crack of the length $2a_0$ in a plane subjected to a uniform stress $\sigma$ perpendicular to the plane of the crack. The stress intensity factor $K$ is given by:

$$k = f(a)\sigma \sqrt{\pi a}$$

(2.3)
Where \( f(a) \) is a geometry dependent function. Integrating the fatigue crack propagation law expressed by equation (2.1) gives:

\[
N - N_0 = \int_a^{a_0} \frac{da}{C(\Delta K)^m} \tag{2.4}
\]

where \( N_0 \) is the number of load cycles corresponding to the half crack length \( a_0 \). Introducing the stress intensity factor range \( \Delta K \), where \( K \) is given from equation (2.2), into equation (2.1) results in:

\[
N - N_0 = \int_a^{a_0} \frac{da}{C[f(a)\Delta\sigma\sqrt{\pi a}]^m} \tag{2.5}
\]

Assuming that the function \( f(a) \) is equal to its initial value \( f(a_0) \) so that

\[
\Delta K = \Delta K_0 \sqrt{\frac{a}{a_0}} , \quad \Delta K_0 = f(a_0)\Delta\sigma\sqrt{\pi a_0} \tag{2.6}
\]

Equation (2.5) gives:

\[
N - N_0 = \frac{2a_0}{(m-2)C(\Delta K_0)^m} \left[ 1 - \left( \frac{a_0}{a} \right)^{\frac{m}{2}} - 1 \right] \text{ for } m \neq 2 . \tag{2.7}
\]

Unstable crack propagation occurs when

\[
K_{max} = K_{IC} = f(a)\sigma_{max}\sqrt{\pi a} \tag{2.8}
\]

from which the critical crack length \( a_0 \) is obtained. Then, the equation 2.5 for \( a = a_0 \) gives the fatigue crack propagation life \( N_p = N_c - N_0 \). Usually, however \( f(a) \) varies with the crack length \( a \) and the integration of equation (2.5) cannot be performed directly, but only through the use of numerical methods.
2.4 Crack Closure Phenomenon in Fatigue Crack Propagation

In the early 1960s, Paris, et al. [2.17, 2.18] demonstrated that fracture mechanics is a useful tool for characterizing crack growth due to fatigue phenomenon. Since that time, the application of fracture mechanics to fatigue problems has become almost routine. There are, however, a number of controversial issues and unanswered questions in this field. The procedures for analyzing fatigue under constant amplitude loading at small scale yielding conditions are fairly well established, although a number of uncertainties remain. The concept of similitude, when it applies, provides the theoretical basis for fracture mechanics. Similitude implies that the crack tip conditions are uniquely defined by a single loading parameter such as the stress intensity factor. Consider a growing crack in the presence of a constant amplitude cyclic stress intensity Figure 2.16. A cyclic plastic zone forms at the crack tip, and the growing crack leaves behind a plastic wake. If the plastic zone is sufficiently small, that is embedded within an elastic singularity zone, the conditions at the crack tip are uniquely defined by the current K value, and the crack growth rate is characterized by $K_{\text{min}}$ and $K_{\text{max}}$. In order for the similitude assumption to be valid, the crack tip of the growing crack needs to be sufficiently far from its initial position, and external boundaries should be remote.

![Figure 2.16](image)

Figure 2.12 Constant amplitude fatigue crack growth under small yielding conditions.

It is convenient to express the functional relationship for crack growth in the following form:
\[
\frac{da}{dN} = f_1(\Delta K, R) \tag{2.9}
\]

Where \( \Delta K = (K_{\text{max}} - K_{\text{min}}) \), \( R = K_{\text{min}}/K_{\text{max}} \)and \( \frac{da}{dN} \) is the crack growth per cycle. The influence of the plastic zone and the plastic wake on crack growth is implicit in equation 2.9, since the size of the plastic zone depends only on \( K_{\text{min}} \) and \( K_{\text{max}} \). A number of expressions for \( f_1 \) function have been proposed, most of which are empirical.

Soon after the Paris law gained wide acceptance as a predictor of fatigue crack growth, many researchers came to the realization that this simple expression was not universally applicable. As Figure 2.9 illustrates, a log-log plot of \( da/dN \) versus \( \Delta K \) is sigmoidal rather than linear when crack growth data are obtained over a sufficiently wide range. Also, the fatigue crack growth rate exhibits a dependence on the R ratio, particularly at both extremes of the crack growth curve. A discovery by Fiber [2.19] provided at least a partial explanation for both the fatigue threshold and R ratio effect.

Elber postulated that crack closure decreased the fatigue crack growth rate by reducing the effective stress intensity range Figure 2.13. When a specimen is cyclically loaded at \( K_{\text{max}} \) and \( K_{\text{min}} \), the crack faces are in contact below \( K_{\text{op}} \), the stress intensity at which the crack opens. Elber assumed that the portion of the cycle that is below \( K_{\text{op}} \) does not contribute to fatigue crack growth.

The definition of the effective stress intensity range is:

\[
\Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{op}} \tag{2.10}
\]

Also the effective stress intensity ratio by Elber is:

\[
U = \frac{\Delta K_{\text{eff}}}{\Delta K} = \frac{K_{\text{max}} - K_{\text{op}}}{K_{\text{max}} - K_{\text{min}}} \tag{2.11}
\]

and consequently a modified version of the equation (2.1) proposed

\[
\frac{da}{dN} = C(\Delta K_{\text{eff}})^m \tag{2.12}
\]
40

Crack closure occurs as a consequence of crack tip plasticity. At the tip of a growing fatigue crack, each loading cycle generates a monotonic plastic zone during increased loading and a much smaller reversed plastic zone during unloading. Approximately the reversed plastic zone size is one-quarter of the size of the monotonic plastic zone. Due to this, there is a residual plastic deformation consisting of monotonically stretched material. As the crack grows, the residual plastic deformation forms a wake of monotonically stretched material along the crack edges. Because the residual deformation is the consequence of tensile loading, the material in the crack edges is elongated normal to the crack surfaces and has to be accommodated by the surrounding elastically stressed material. This is no problem as long as the crack is open, since then the crack edges will simply show a displacement normal to the crack surfaces. However, as the fatigue load decreases, during unloading, the crack will tend to close and the residual deformation becomes important.

2.5 Effect of Residual Stresses on Crack Propagation

Our knowledge of the relationship between residual stress and fatigue strength is perplexed due to the fact that:

- The fatigue strength depends greatly on the condition of the surface. The effect of residual stress is overshadowed by such major factors as weld geometry and surface irregularities.
- A fatigue crack may initiate in a region containing tensile residual stresses. The rate
of crack growth may be increased due to the existence of tensile residual stresses. However, when the crack grows and enters regions containing compressive residual stresses, the rate of the crack growth may be reduced. As a result, it is questionable whether or not the total effect of residual stresses on the overall crack growth is significant.

- When residual stresses are altered by a heat treatment such as peening, the metallurgical and mechanical properties of the metal are also changed. A schematic presentation of the stress field behind and in front of a crack tip under cyclic loading without welding residual stresses is illustrated in Figure 2.14
Figure 2.14 Formation of reverse plastic zone during cyclic loading, [2.20].

How residual stresses actually affect the plastic zone showed in Figure 2.14 and subsequently the fatigue strength of a welded structure is still a matter of debate. Some researchers have reported that the fatigue strength increased when specimens had compressive residual stresses, especially on the specimen surfaces, others believe that residual stresses have only a negligible effect on the
fatigue strength of the weld elements. It has been suggested that in a good weld residual stresses can be ignored. Also it has been suggested that geometry affects fatigue behavior much more than residual stresses. But others researchers feel that there is significant evidence that residual stresses affect the fatigue strength. Munse [2.21] summarizes as follows:

"On the basis of the available data it is believed that the effects of residual stresses may differ from one instance to another, depending upon the materials and geometry of the members, the state of stress, the magnitude of applied stress, the type of stress cycle and perhaps other factors. Many of the investigations designed to evaluate the effects of residual stress have included tests of members that have been subjected to various stress relief heat treatments. The changes in fatigue behavior resulting from these heat treatments, in some cases, have been negligible, while in other investigations, the various stress-relief treatments have produced an increase in fatigue strength of as much as twenty percent. Since it is impossible to carry out a heat treatment for stress relief without altering the metallurgical and mechanical properties of weldment, the question always arises as to whether benefits are derived from the reduction of residual stresses or from the improved properties in other respects."

2.6 Design of integral structure

According to NASA’s research, “About a third of the airlines’ direct operating cost (DOC) of an airplane is associated with the manufacturing cost, which is probably the most critical competitive parameter with regard to market share [2.22]. It means that it is an effective way to cut down the manufacturing cost to reduce the acquisition cost of an aircraft. The skin-stringers riveted structures have been used in aircraft fuselage for more than 60 years. These kind of riveted structures have advantages in damage tolerance performance and also fail-safe, since stringers gives another path for load passing, which delays the speed of crack growth. But this kind of design makes it difficult to reduce in cost significantly because they are highly refined and mature with associated construction details and fabrication processes. Nevertheless, metallic structure is well proved, and it will likely retain extensive metallic production capability and skills in the foreseeable future. Hence, the conception of designing renewed large integral metallic skin-stringer panels for aircraft fuselage for low acquisition cost and the emergence of high speed
machining is imminent. A typical integral structure made by NASA’s ISA program shows in Figure (2.15).

![Figure 2.15 Typical integral fuselage.](image)

The results were exciting when machined integral structures were taken into Boeing 747 fuselage. It was found to be superior in terms of part count and cost, and almost equivalent in terms of weight when compared with riveted structure. These results are summarized in Table 2-1 [2.23].

<table>
<thead>
<tr>
<th>Factor</th>
<th>Riveted Panel</th>
<th>Integral Panel</th>
<th>Integral Change From Riveted</th>
<th>Target Savings Over Riveted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of parts</td>
<td>78</td>
<td>7</td>
<td>91% reduction</td>
<td>50%</td>
</tr>
<tr>
<td>Weight</td>
<td>179 pounds</td>
<td>186 pounds</td>
<td>4% increase</td>
<td>Neutral</td>
</tr>
<tr>
<td>Estimated Cost</td>
<td>$33.000</td>
<td>$14.000</td>
<td>58% reduction</td>
<td>25%</td>
</tr>
</tbody>
</table>
2.7 Comparison of riveted and integral structures

It is necessary to investigate the integral panels in details in order to ascertain the possible high benefits over riveted panels. Figure (2.16) below gives the difference between conventional riveted stringer fuselage panel and the new integral skin-stringer fuselage panel. Figure (2.17) describes the riveted stringer panel and the integral skin-stringer panel.

![Figure (2.16) Structure of riveted panel and integral fuselage panel.](image1)

![Figure (2.17) Riveted stringer panel and integral stringer panel.](image2)
[2.24] compared the damage tolerance behavior of integrally skin-stringer structures and riveted structures, and gave the pros and cons as follow: For riveted stringer panel, the pro is offering fail safety for the hard of crack going to the stiffener. The cons are causing premature initiation of fatigue cracks, thousands of fasteners to be used and the fact that they are difficult to manufacture and inspect. For integral stringer panel, the pro are reducing part count and structural complexity, automated processing and improving visual inspection capability. The cons are lacking of redundant structural members, lacking of damage tolerance behavior and increasing crack growth rates in heat affected zones.

2.8 Improvement of Integral Structures

In order to optimize the damage tolerance performance of integral metallic structures, two particular aspects should be considered. The first one is developing new kinds of materials with a better fracture toughness property [2.25]. Although the 7000 series aluminum alloys have sensational mechanical performance, toughness sharp reduction at low temperatures which is especially dangerous for the integral metallic structures limits its use. Since 2000 series aluminum alloys are not so sensitive to very low temperature, they can be exploited to overcome the disadvantage. Another one is designing or optimizing structures. In recent years, researchers analyzed many different methods for the structure design optimization. It is an effective way to save the time and money for the prototype building through the development of methods to simulate the crack growth behavior of the components. Retarders of crack growth, which are bonded to integral metallic panels, were investigated in order to overcome the lack of a fail safety performance. In order to create a failsafe design feature, a hybrid structure bonding two different materials together is created in critical zone [2.26]. These bonded straps still have some disadvantages, even though they have advantages in delaying the fatigue crack growth. Another way for optimization is to reduce crack growth speed in the integral panels through the investigation of the optimized shapes. Stringers which play important roles in the damage tolerance behavior of integral panels are the most promising fields to analysis [2.27]. According to the research, the stress intensity factor (SIF) decreases when the crack approaches a stiffener and it increases when the stiffener has been crossed. The overall result is the crack grows slow, because the crack growth depends on SIF variation. Besides, stiffeners increase T-stress, which may cause
crack turning. Hence, it is important to build an effective model to describe the SIF evolution during the crossing of the stiffener, in an accurate way.
CHAPTER THREE: FRACTURE MECHANICS AND XFEM (Extended Finite Element Method) concepts

3.1 Introduction:

This chapter introduces briefly the analysis methods for SIF (stress intensity factor) calculation and crack growth life prediction for integral stiffened panels. There are three types of loading that a crack can experience, as Figure 3.1 illustrates. Mode I loading, where the principal load is applied normal to the crack plane, tends to open the crack. Mode II corresponds to in-plane shear loading and tends to slide one crack face with respect to the other. Mode III refers to out-of-plane shear. A cracked body can be loaded in any one of these modes, or a combination of two or three modes. All the SIF obtained by using extended finite element method (XFEM).

![Figure 3-1 Fracture modes [3.1]](image)

For linear elastic Materials, Griffith’s approach says that a crack extends if the thermodynamic crack driving force, characterized by the energy release rate $G$, becomes equal or larger than the crack growth resistance, $R$ (Griffith, 1921) [3.1][3.2] where as in 1956 Irwin proposed an energy
approach for fracture that is essentially equivalent to the Griffith model, except that Irwin’s approach is in a form that is more convenient for solving engineering problems. He postulates that a crack grows when the crack tip stress intensity factor $K$ reaches a critical value $K_c$. The Griffith and Irwin criteria are equivalent for linear elastic materials, since energy release rate and stress intensity factor are related. The assumptions taken in LEFM analysis is listed below [3.1]:

1. A sharp crack or flaw of similar nature already exists; the analysis deals with the propagation of the crack from the early stages.
2. The material is linearly elastic.
3. The material is Isotropic.
4. The size of the plastic zone near the crack-tip is small compared to the dimensions of the crack.
5. The analysis is applicable to near-tip region.

Linear Elastic fracture mechanics (LEFM) is valid only as long as non-linear material nonlinear material deformation is confined to a small region surrounding the crack tip. In many materials, it is virtually impossible to characterize the fracture behavior with LEFM, and an alternative fracture mechanics model is required. Elastic-Plastic Fracture Mechanics applies to materials that exhibit time dependent, nonlinear behavior (i.e. plastic deformation) [3.2]. For crack growth in elastic-plastic materials under large scale or general yielding conditions, the common approach is to use criteria based on the crack tip opening displacement (CTOD) by Wells in 1963, Rice’s $J$-integral in 1968 [3.1] and the energy dissipation rate by Turner and also Turner Kolednik in 1994.

### 3.2 Classical fracture criteria and parameters:

#### 3.2.1 The Stress Intensity Factor.

A major activity in the design process based on fracture mechanics is the determination of the Stress Intensity Factor (in the following simply SIF). In the following sections, some of the pertinent analytical and numerical methods are discussed.
-Analytical determination of SIF

SIF can be coupled by an analytical approach in some relevant cases. Incase of an infinite plate with a central crack of length ‘2a’, under remote stress $\sigma_0$ the calculating of SIF is as follow: (see Fig. 3.2)

![Diagram of an infinite plate with a central elliptic crack](image)

**Fig 3.2: An infinite plate with a central elliptic crack [3.3]**

Because the linear elastic fracture mechanics approach is based on elasticity, one can determine the effects of more than one type of loading on the crack tip stress field by linearly adding the SIF due to each type of loading. The process of adding SIF solutions for the same geometry is sometimes referred to as “principle of superposition”. The only constraint on the summation process is that the SIF must be associated with the same structural geometry, including crack geometry. Thus, for the geometries shown in Fig. 3.3 the equation is as follow:
Fig 3.3: The geometries of superposition of K expression [3.4]

\[ K_{IA} + K_{IB} = \sigma \sqrt{\pi a} \] 3-1

\[ K_{IA} + 0 = \sigma \sqrt{\pi a} \] 3-2

\[ K_{IA} = \sigma \sqrt{\pi a} \] 3-3

The stress distribution around the tip in mode I is described by Westergaard [3.5] as follow (Fig 3.4).
Fig 3.4: The stress distribution around the crack tip. [3.6]

\[ \sigma_t = \frac{K_I}{\sqrt{2\pi r}} f(\theta) \] 3-4

\[ \sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} (1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2}) + \ldots \ldots \] 3-5

\[ \sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} (1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2}) + \ldots \ldots \] 3-6

\[ \tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \sin \frac{\theta}{2} \ldots \ldots \] 3-7

For distances close to the crack tip \((r \leq 0.1a)\), the second and higher order terms indicated by dots may be neglected. The \(I\) subscript is used to denote the crack opening mode, but similar relations apply in modes II and III. Above equations show three factors relevant to depict the
stress state near the crack tip: denominator $\sqrt{2\pi r}$ shows the singular nature of the stress distribution; $\sigma$ approaches infinity as the crack tip is approached, with $a/\sqrt{r}$ dependency. Depend on angle $\theta$; it can be separated if a suitable factor is introduced. $f_x = \cos \theta/2 \cdot (1 - \sin \theta/2 \sin 3\theta/2) + \ldots$. Factor $K_I$ contains the dependence on applied stress $\sigma$, the crack length $a$, and the specimen geometry. The $K_I$ factor gives the overall intensity of the stress distribution, hence its name. For the specific case of a central crack of width $2a$ or an edge crack of length $2a$ in a large sheet, $K_I = \sigma a \sqrt{\pi a}$ and $K_I = 1.12 \sigma a \sqrt{\pi a}$ for an edge crack of length ‘a’ in the edge of a large sheet. Expressions for $K_I$ for some additional geometry are given in Table 3.1. The literature [3.7] contain expressions for $K$ for a large number of crack and loading geometries, and both numerical and experimental procedures exist for determining the stress intensity factor is specific actual geometries.

<table>
<thead>
<tr>
<th>Type of Crack</th>
<th>Stress Intensity Factor, $K_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center crack, length $2a$, in an infinite plate</td>
<td>$\sigma a \sqrt{\pi a}$</td>
</tr>
<tr>
<td>Edge crack, length $a$, in a semi infinite plate</td>
<td>$1.12 \sigma a \sqrt{\pi a}$</td>
</tr>
<tr>
<td>Central penny shaped crack, radius $a$, in infinite body</td>
<td>$K_I = 2 \sigma a \sqrt{\pi/a}$</td>
</tr>
<tr>
<td>Center crack, length $2a$ in plate of width $W$</td>
<td>$\sigma a \sqrt{W \tan (\frac{\pi a}{W})}$</td>
</tr>
<tr>
<td>Two symmetrical edge cracks, each length $a$, in plate of total width $W$</td>
<td>$\sigma a \sqrt{W \tan (\frac{\pi a}{W})} + 0.1 \sin (\frac{2\pi a}{W})$</td>
</tr>
</tbody>
</table>

Table 3.1: Expressions of $K_I$ for different geometries

These SIF’s are used in design and analysis by arguing that material can withstand crack tip stresses up to a critical value of stress intensity, termed $K_{IC}$, beyond which the crack propagates very fast. This critical SIF is then a measure of material toughness. The failure stress $\sigma_f$ is then related to the crack length $a$ and the fracture toughness by:
\[ \sigma_r = \frac{K_{Ic}}{\alpha \sqrt{a}} \]

Where \( \alpha \) is a geometrical parameter equal to 1 for edge cracks and generally on the order of unity for other boundary conditions. Expressions for \( \alpha \) are tabulated for a wide variety of specimen and crack geometries.

Typical values of \( G_{Ic} \) and \( K_{Ic} \) for various materials are listed in Table 3.2 [3.8, 3.9 and 3.10], and it is seen that they vary over a very wide range from material to material. Some polymers can be very tough, especially when rated on per-pound bases, but steel alloys are hard to beat in terms of absolute resistance to crack propagation.

<table>
<thead>
<tr>
<th>Material type</th>
<th>Material</th>
<th>( K_{Ic} ) (MPa ( \sqrt{m} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal</td>
<td>Aluminum alloy (7075)</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>Steel alloy (4340)</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>Titanium alloy</td>
<td>44–66</td>
</tr>
<tr>
<td></td>
<td>Aluminum</td>
<td>14–28</td>
</tr>
<tr>
<td>Ceramic</td>
<td>Aluminium oxide</td>
<td>3–5</td>
</tr>
<tr>
<td></td>
<td>Silicon carbide</td>
<td>3–5</td>
</tr>
<tr>
<td></td>
<td>Soda-lime glass</td>
<td>0.7–0.8</td>
</tr>
<tr>
<td></td>
<td>Concrete</td>
<td>0.2–1.4</td>
</tr>
<tr>
<td>Polymer</td>
<td>Polymethyl methacrylate</td>
<td>0.7–1.6</td>
</tr>
<tr>
<td></td>
<td>Polystyrene</td>
<td>0.7–1.1</td>
</tr>
<tr>
<td>Composite</td>
<td>Mullite-fibre composite</td>
<td>1.8–3.3</td>
</tr>
<tr>
<td></td>
<td>Silica aerogels</td>
<td>0.0008–0.0048</td>
</tr>
</tbody>
</table>

Table 3.2: Typical values of \( K_{Ic} \) for various materials

3.2.2. The Energy Release Rate

By the analysis of the energy balance, the energy release rate, denoted as \( G \), was introduced. It is defined by the energy necessary to make the crack fronts extending the crack length by a
unit length. It corresponds to the decrease of the total potential energy $W_{pot}$ of the cracked body, when it passes from an initial configuration with a given crack length, to another configuration where the crack is increased by a unit of length “da” [3.11]:

$$G = -\frac{dW_{pot}}{da}$$  \hspace{1cm} 3-9

$$W_{pot} = W_{e} - W_{ext}$$  \hspace{1cm} 3-10

Where: $W_{ext}$ is the work of external forces and $W_{pot}$ is the total potential energy of crack body and $W_{e}$ is strain energy of structure.

Using the stress field in the singular zone, one can relate $G$ to the stress intensity factors:

$$G = \frac{(K_{I}^{2}+K_{II}^{2})}{E'} + \frac{K_{III}^{2}}{2\mu}$$  \hspace{1cm} 3-11

With $E' = E$ in plane stress

$$E' = E/(1 - \nu^2)$$ in plane strain

$\mu$ : shear modulus
3.2.3 The J-Integral.

J-integral is a parameter to deal with Non-linear fracture problem, which is proposed by Rice [3.11]. J-integral is less dependent on crack tip stress singularity for it is based on the concept of conservation of energy, which means there is no need to do special treatment on the mesh around crack tip. As shown in Figure 3.5, the equation of J-integral is

\[ J = \int_{\Gamma} \left( W \, dx_2 - T_i \frac{\partial u_i}{\partial x_j} \right) \, ds \]

Figure 3-5 Counter clockwise loop around the crack tip.

Where \( w \) is the strain energy density, \( T_i \) is the traction vector, \( u_i \) is the displacement vector, \( ds \) is an element of arc along the integration contour.
• **Relation between J and K.**

In LEFM, the stress and displacement components at the crack tip are known as a function of the position relative to the crack tip. For multi-mode loading, they are characterized by the stress intensity factors $K_I$, $K_{II}$ and $K_{III}$. Because the J-integral is path-independent, the integration path can be chosen to be a circle with the crack tip as its center. Integration over this circular path reveals that the J-integral is related to the SIF. For Mode I loading of the crack, it follows immediately that the J-integral is equivalent to the energy release rate $G$. This means that J-integral can be used in the crack growth criteria of LEFM as a replacement for $K$ and $G$ [3.11].

\[
\text{Plane stress } J = \frac{1}{E} K_I^2
\]

\[
\text{Plane strain } J = \frac{(1-v^2)}{E} K_I^2
\]

• **J-integral crack growth criterion**

J-integral can replace the energy release rate in LEFM and is related to the SIF. When the material behavior is described by the general Ramberg-Osgood relation [3.12], the J integral characterizes the stress at the crack tip. It is thus obvious that it can be used in a crack growth criterion. Calculation of its value is easily done, due to the fact that the integration path can be chosen arbitrarily. Critical values have to be measured according to normalized experiments

\[
J = J_c
\]
3.2.4 Crack Tip Opening Displacement (CTOD).

In LEFM the displacement of material points in the region around the crack tip can be calculated. With the crack along the x-axis, the displacement $u_y$ in y-direction is known as a function of $r$ (distance) and $\theta$ (angle), both for plane stress and plane strain. Displacement of points at the upper crack surface results for $\theta = \pi$ and can be expressed in the coordinate x, by taking:

$$r = a - x \quad 3-16$$

Where $a$ is the half crack length. The origin of this xy-coordinate system is at the crackcenter. The crack opening (displacement) (COD) $\delta$ is two times this displacement. It can be easily appreciated that the opening at the crack tip (CTOD), $\delta_t$, is zero [3.13].

$$u_y = \frac{\sigma\sqrt{\pi a}}{2\mu} \sqrt{\frac{r}{2\pi}} \left[ \sin \left( \frac{\theta}{2} \right) \left( k + 1 - 2\cos^2 \left( \frac{\theta}{2} \right) \right) \right] \quad 3-17$$

Displacement in crack plane $\theta = \pi$; $r = a - x$:

$$u_y = \frac{(1+\nu)(k+1)}{E} \frac{\sigma}{2} \sqrt{2a(a-x)} \quad 3-18$$

Crack Opening Displacement (COD):

$$\delta x = 2u_y(x) = \frac{(1+\nu)(k+1)}{E} \sigma \sqrt{2a(a-x)} \quad 3-19$$

Crack Tip Opening Displacement (CTOD):

$$\delta t = \delta(x = a) = 0 \quad 3-20$$
This CTOD can be used in a crack growth criterion (Fig 3.6), when plasticity at the crack tip is taken into account and the actual crack length is replaced by the effective crack length.

3.3 Numerical tools.

There are many numerical approaches currently available to solve the problems concerning LEFM and to calculate the SIF. Research activity in this domain has produced a very large number of papers and it would be extremely difficult, and perhaps useless in the frame of this thesis, to extensively review all the literature on this subject. Therefore, following state of the art only briefly includes some of the basic references and proposes a classification of proposed methods by defining categories.
a) **The Finite Element Method (FEM).**

Usually, displacement-type finite elements (based on the virtual work principle) are widely used. According to the so-called “direct approach”, the SIF are deduced from the displacement field, this is the case of the Crack Opening Displacement method (COD). In the “energy approach” which is generally more precise, the SIF are deduced from the energy distribution in the proximity of the crack tip, either from the energy release as in the method of the Virtual Crack Extension or from the J-integral as in the Equivalent Domain Integral Method [3.14]

b) **The Boundary Elements Method (BEM).**

In this method, only the boundaries of the solid are discretized. The partial differential equations of the Theory of Elasticity are transformed into integral equations on the boundaries of the domain. Basically, the primary unknowns of the numerical problem are the displacements. This is the case for the “crack Green’s function method”, the “displacement discontinuity method” and the “sub regions method”. In dual method has been developed, which the surface tractions as primary unknowns [3.15].

c) **The Mesh less method.**

This method has been applied to fracture mechanics since 1994 and, subsequently, different improvements have been introduced, for example to couple this approach with the finite element method, to ensure the continuity of displacements in the vicinity of crack and improve the representation of the singularity at the crack tip, by using an arbitrary Lagrangian-Eulerian formulation to enrich the displacement approximation near the crack tip or to enrich the weighting functions [3.16].

d) **The Extended Finite Element Method.**

The Extended Finite Element Method (XFEM) allows some discontinuities in the assumed displacement field. Discontinuities can be due to the presence of cracks and do not have to coincide exactly with the finite element edges: they can be located anywhere in the domain independently
of the finite element mesh [3.17]. This approach is extremely used in the recent literature of fracture mechanics and is highly supported by the ABAQUS © code.

3.3.1 The Finite Element Method.

Many issues of structural integrity can be cast as problem of linear elastic fracture mechanics (LEFM). These can include fatigue crack propagation and life prediction, other types of sub-critical crack growth, residual strength estimation, and brittle fracture. In these and other related problems, it is essential to be able to predict the onset of crack growth, and its rate, shape, and stability. The finite element method, as performed within modern high-performance and low cost computing environments, is a natural tool for analyzing such LEFM problems.

A) Singular finite elements.

A fundamental difficulty when modeling linear elastic fracture mechanics (LEFM) problems through FEM is that polynomial basis functions used for most conventional elements cannot represent the singular crack-tip stress and strain fields predicted by the theory. This means that mesh doesn’t assure the numerical convergence to the theoretical solution, although it is highly refined around the crack tip.

A significant improvement in the use of FEM for LEFM problems was the simultaneous, and independent, development of the quarter-point element. Crack tip displacement, stress and strain fields are modeled by standard quadratic order isoparametric finite elements if one simply moves the elements mid-side node to the position one quarter of the way from the crack tip to the far end of the element. This procedure introduces a singularity into the mapping between the element’s parametric coordinate space and the Cartesian space [3.18]. The quadratic quarter-point element is illustrated in Fig 3.7.
The introduction of quarter-point elements was a significant milestone in the development of finite element procedures for LEFM. With these elements standard and widely available, finite element programs can be used to model crack tip fields accurately, with only minimal preprocessing required.
B) Extracting SIF and Energy Release Rate from FEM.

Under LEFM assumptions, the stress, strain and displacement fields in the near crack-tip region are determined by the SIF. Therefore, extraction of accurate SIF is a fundamental task of FEM modeling. [3.19]. There are four techniques very often applied: displacement correlation, virtual cracks extension, modified crack closure integral and the J-integral; these techniques look more accurate and simple. It is worthy motivated that techniques for extracting SIF’s fall into one of two categories above-mentioned. Some belong the direct approaches, which correlate the SIF’s with FEM results directly and energy approach, which compute the energy release rate. In general, the energy approaches are more accurate and should be used preferentially. However, the direct approaches are especially useful as a check on energy approaches because expressions are simple enough to handle the calculations. A brief description of four mentioned techniques follow:

**Generalized Displacement correlation method** is one of the simplest first techniques proposed to extract SIF’s from the FEM displacements for a node of the mesh, by substituting directly displacement value into the analytical expressions for near-tip displacement, after subtracting the displacements of the crack tip. Usually, a node on the crack face where the displacements will be greatest is selected and thus the relative error in the displacements is expected to be smallest. A generalized form displacement correlation method (GDC) can use any linear or quadratic finite element type with homogeneous meshing without local refinement. These two features are critical for modeling dynamic fracture propagation problems where locations of fractures are not known a priori. Because regular finite elements’ shape functions do not include the square-root terms, which are required for accurately representing the near-tip displacement field, the GDC method is enriched via a correction multiplier term. The proposed method using quadratic elements is accurate for mode-I and mode-II fracturing, including for very coarse meshes. An alternative formulation using linear elements is also demonstrated to be accurate for mode-I fracturing, and acceptable mode-II results for most engineering applications can be obtained with appropriate mesh resolution, which remains considerably less than that required by most other methods for estimating stress intensities [3.20]. The configuration for this simple approach is shown in Fig 3.8.
Fig 3.8: Possible sample point location for simple displacement correlation

**The virtual crack extension method** is an energy approach that computes the rate of change in the total potential energy of a system for a small extension of the crack. Under LEFM assumption, this is equal to the energy release rate. In general the virtual extension crack is more accurate than the displacement correlation approach for a given finite element mesh. However, as originally proposed, only a total energy release rate is computed. It is not separated for the three modes of fracture \[3.21\].

**The modified crack closure integral (MCCI) technique** was originally proposed by Rybicki and Kanninen \[3.22\], They observed that Irwin’s crack closure integral could be used as computational tool (Fig 3.9). Release the energy release rate to the crack-tip stress and displacement fields for a small crack increment.
Fig 3.9: Crack-tip stress and displacement fields used in Irwin crack closure integral

The MCCI procedure has been extended for use with higher order element. Of particular interest is its formulation for quadratic-point elements is insensitive. These elements express the crack-tip displacement and stress fields in terms of second order polynomials that were consistent with the quarter-point behavior.

In general, for a given mesh the MCCI technique yields SIF’s that are more accurate than the displacement correlation approach, but less accurate than the J-integral approach. However, it gives surprisingly accurate results for its simplicity and requires nodal forces and displacements only, which are standard outputs from many finite element programs.

The J-integral is well-known parameter of nonlinear fracture mechanics. Under linear elastic material assumptions, the J-integral can be interpreted as being equivalent to the energy release rate, G. In its original formulation, it relates the energy release rate of a two dimensional body to a contour integral. The contour integral in the simple form can be shown to be path-independent providing there are no body forces inside the integration area, there are no tractions on the crack surface and the material behavior is elastic [3.23].
Life prediction Methods The fatigue life as a whole can be divided into three parts: crack initiation, crack propagation, and final failure. Several conventional fatigue analysis methods are used in first phase life estimation such as the S-N curve approach and detail fatigue-rating approach. A small crack is assumed in the beginning of fatigue life calculation. Although the small flaw may not be fracture critical under static loads, it will gradually increase under cyclic loads. Therefore, the ability of the prediction of a component under cyclic loads becomes particularly important. During the crack propagation process, stress intensity factor plays a decisive role. It is assumed that the crack growth rate is determined by the stress intensity factor range, and different cracks have same rate of propagation if they have the same stress intensity factor. Thus, the crack propagation rate, $da/dN$ has the relationship with stress intensity factor range,

$$\Delta K = K_{\text{max}} - K_{\text{min}}$$ 3-21

$$da / dN = f(\Delta K)$$ 3-22

Paris Equation Paris, etc were the first to find the relationship between the crack growth rate and the SIF, and began to compare it with test data [3.24]. They gave the equation in the following form:

$$da / dN = C(\Delta K)^n$$ 3-23

This is Paris law, where $C$ and $n$ were constants related to the material.

Forman’s Equation Forman’s law is also a kind of life prediction method, which considers the mean stress effect of a fatigue stress cycle [3.25]. The equation is in the following form:

$$\frac{da}{dN} = \frac{C(\Delta K)^n}{(1-R)K_c - \Delta K}$$ 3-24

Where $R=S_{\text{min}}/S_{\text{max}}$ reflects the mean stress effect. $K_c$ is the fracture toughness which describes the effect when $K_I$ near to $K_{IC}$.

As the result of fatigue testing experience, $\Delta K_{th}$ is also related to the stress ratio and material property. Hence, Forman’s equation can be modified as follow:
\[
\frac{da}{dN} = \frac{C(\Delta K - \Delta K_a)^n}{(1 - R)K_c - \Delta K}
\]

C) Available tools and software.

In the FEM, the structure is subdivided into discrete elements. Different Element types can be used to cover the problem. Elements are connected at node, where continuity of displacement field is imposed. Displacements at nodes depend on the element stiffness and computational of the nodal forces. For structural problems, numerical solution consists of computing nodal displacements. Stress and strain distributions throughout the body, as well as the crack parameters such as SIF, can be inferred from the nodal displacements. A number of commercial FEM packages have the ability of crack modeling and performing the fracture mechanics calculations. There is also some noncommercial code, as the FRANC2D, which is developed by the Cornell University, being surprisingly easy to learn and offering many capabilities. Finite element analysis can be carried out by several available software like ABAQUS, ANSYS, and LS-DYNA etc. These software’s are user friendly and give a wide range of analysis options. Static, dynamic, fluid, thermal and electromechanical problems can be analyzed by means of those codes.

In this thesis, the ABAQUS© was used, it can solve linear and nonlinear problems. It was designed to be able to investigate many links of nonlinearities such as geometrical material or multi-physic domains. Some specialized modules allow investigation of several behavior of material in presence of plasticity, buckling, electromechanical coupling and even fracture. Numerical tools are evaluated to solve nonlinear problems by an automatic updating of the set-up to assure the numerical convergence and an accurate result.

3.3.2 Extended Finite Element Method (XFEM)

The standard finite element method (FEM) provides substantial advantages in dealing with continuous field problems. However, for discontinuous field problems, it is computationally expensive to obtain accurate solutions with polynomial approximations. Alignment of mesh with
discontinuity becomes a major difficulty when treating problems with evolving discontinuities where the mesh must be regenerated at each step, i.e. re-meshing is needed continuously [3.26]. Modeling of cracks in structures and especially involving cracks requires the FEM mesh to conform the geometry of the crack and hence needs to be updated each time as the crack grows. This not only computationally costly and cumbersome but also results in loss of accuracy as the data is mapped from old mesh to the new mesh [3.27].

A re-meshing technique is traditionally used for modeling cracks within the framework of finite element method where a re-meshing is done near the crack to align the element edges with the crack faces [3.28].

The Extended finite element method (XFEM), also known as generalized finite element method (GFEM) or Partition of unity method (PUM) is a numerical technique that extends the classical finite element method (FEM) approach by extending the solution space for solutions to different equations with discontinuous functions. It was first introduced by Bolyteschko and Black[3.29]. The extended finite element method (XFEM) has proved to be a competent mathematical tool [3.30] since it is an extension of partition of unity; allows the presence of discontinuities in an element by enriching degrees of freedom with special displacement functions. [3.31]

![Figure 3.10: Mesh discretization in XFEM (left) and FEM (right)[3.34].](image)

In comparison to the classical finite element method, the XFEM provides significant benefits in the numerical modeling of crack propagation. The traditional formulation of the FEM, the existence of crack is modeled by requiring the crack to follow element edges. In contrast, the crack

68
geometry in the XFEM need not to be aligned with the element edges, which provides flexibility and versatility in modeling.

The Extended Finite Element Method (XFEM) can dramatically simplify the solution of many problems in material modeling such as the propagation of cracks, the evolution of dislocations, the modeling of grain boundaries and the evolution of phase boundaries [3.32].

The method is based on enrollment of the FE model with additional degrees of freedom (DOF) that are tied to the nodes of the elements intersected by the crack [3.32][3.34]. In this manner, the discontinuity is included in the numerical model without modifying the discretization, as the mesh is generated without taking into account the presence of the crack. Therefore, only a single mesh is needed for any crack length and orientation. In addition, nodes surrounding the crack tip are enriched with DOFs associated with functions that reproduce the asymptotic LEFM fields. This enables the modeling of the crack discontinuity with in the crack tip and substantially increases the accuracy in the computation of the stress intensity factors (SIFs).

3.3.3 Partition of Unity Finite Element Method, PUFEM

Partition of unity is a set R of continuous functions from X to the interval [0, 1] such that for every point $x \in X$

There is a neighborhood of x where all but one finite number of the functions of R are 0,

The sum of all the function values at x is 1, $\sum_{i=1}^{n} f_i(x) = 1$

Partitions of unity are useful because they often allow extending local constructions to the whole space. They are also important in the interpolation of data, in signal processing, and the theory of spline functions [3.34].

To improve a finite element approximation, the enrichment procedure may be applied. In other words, the accuracy of solution can be improved by including the analytical solution of the problem in the finite element formulation. In fracture mechanics problem, if the analytical fracture tip
solution can be added to the framework of the finite element discretization, predicting fracture tip fields may be improved. This will results in increase in the number of degrees of freedom.

The partition of unity finite element method (PUFEM) using the concept of enrichment functions along with the partition of unity property, can help to obtain the following approximation of the displacement within a finite element.

\[ u^h(x) = \sum_{j=1}^{m} N_j(x)(u_j + \sum_{i=1}^{n} P_i(x)a_{ji}) \]  

Where, \( P_i(x) \) are the enrichment functions and \( a_{ji} \) are the additional unknowns or degrees of freedom associated to the enriched solution \( m \) and \( n \) are the total number of nodes of finite elements and the number of enrichment functions \( P_i \).

### 3.3.4 Enrichment Function

In two-dimensional problems, fracture modeling is characterized using of two different types of enrichment functions:

1. The Heaviside Function.

For the elements completely cut by the fracture, The Heaviside function \( H(x) \) is applied for enrichment. The splitting of the element by the fracture results in a jump in the displacement field and the Heaviside function provides a simple mathematical approach to model this kind of behavior.
Figure 3.11: Evaluation of Heaviside function

For a continuous curve $\Gamma$ representing a fracture within the deformable body $\Omega$, we can consider a point $x(x, y)$ in $\Omega$. The objective is to determine the position of this point with respect to the fracture location. If the closest point belonging to $\Gamma$ is $x(x_f, y_f)$ and the outward normal vector to $\Gamma$ is $n$, the Heaviside function might be defined as follows:

$$H(x, y) = \begin{cases} 
1 & \text{for } (x - x_f).n > 0 \\
-1 & \text{for } (x - x_f).n < 0 
\end{cases}$$

This function introduces the discontinuity across the fracture faces.

2. Asymptotic Near-Tip Field Functions.

For those elements that are not completely fractured and containing fracture tip, the Heaviside function cannot be used to approximate the displacement field in the entire element. For the fracture tip, the enrichment functions originally introduced by Fleming for use in the element free Galerkin Method. These four functions describe the fracture tip displacement field. The first function is discontinuous at the fracture tip.
In this formulation $r, \theta$ are polar coordinates defined at the fracture tip. The above functions can reproduce the asymptotic mode I and mode II displacement fields in LEFM, which represent the near-tip singular behavior in strains and stresses. These functions significantly improve the accuracy of calculation of $K_I$ and $K_{II}$.

The term $\sqrt{r} \sin \left( \frac{\theta}{2} \right)$ is discontinuous and therefore can represent the discontinuous behavior at the fracture tip. The remaining three functions are used to enhance approximation of the solution in the neighborhood of the fracture tip.

The circled nodes are the nodes of elements completely cut by the fracture and therefore enriched with Heaviside function. The nodes with Green Square are containing fracture tip and are enriched by fracture tip special function mentioned in equation above.

![Enriched nodes in the XFEM](image)

**Figure 3.12:** Enriched nodes in the XFEM. Circles: nodes with 2 additional DOFs. Squares: nodes with 8 additional DOFs [3.33].
Generally, for the purpose of fracture analysis, the enrichment functions typically consist of the near tip asymptotic functions that capture the singularity around the crack tip and a discontinuous function that represents the jump in displacement across the crack surfaces. The approximation for a displacement vector function with the partition of unity enrichment \[3.29\][3.31][3.32][3.35].

\[
\mathbf{u} = \sum_{i=1}^{N} N_i(X) \left[ \mathbf{u}_1 + H(X) \mathbf{a}_1 + \sum_{\alpha=1}^{4} F_{\alpha}(X) \mathbf{b}_{\alpha} \right]
\]  

Where \( \mathbf{u} \) is the displacement vector.

\( N_i(x) \) is the shape functions which applies to all nodes in the model

\( H(x) \) is the jump function and applies to nodes whose shape function support is cut by the crack interior.

\( \mathbf{a}_1 \) is the nodal enriched degrees of freedom vector.

\( F_{\alpha}(x) \) is the asymptotic crack tip functions.

\( \mathbf{b}_{\alpha} \) is the nodal enriched degree of freedom vector.

The third term in the right side is applies to nodes shape function support is cut by the crack tip.

### 3.3.5 Level set Method for Modeling Discontinuities

In some cases, numerical simulations include moving objects, such as curves and surfaces on a fixed grid. This kind of modeling and tracking is difficult and requires complex mathematical procedure. The Level set Method (LSM) is a numerical technique that can help solving these difficulties. The key point in this method is to represent moving object as a zero level set function. To fully characterize a fracture, two different level set functions are defined:

1. A normal function, \( \varphi(x) \)
2. A tangential function, \( \psi(x) \)
Figure 3.13: Construction of level set functions [3.32]

For the evaluation of the signed distance functions, assume $\Gamma_c$ be the fracture surface and $x$ the point we want to evaluate the $\phi(x)$ function. The normal level set function can be defined as

$$\phi(x) = (x - \bar{x}) \cdot n$$

Where $\bar{x}$ and $n$ are defined previously.

The tangential level set function $\psi(x)$ is computed by finding the minimum signed distance to the normal at the fracture tip. In case of an interior fracture, two different functions can be applied. However, a unique tangential level set function can be defined as.

$$\Psi(x) = \max (\Psi_1(x), \Psi_2(x)) \quad 3-30$$

In conclusion, referring the figure above it may be written as follows:

$$\begin{cases} 
\text{for } x \in \Gamma_{cr} \ (x = 0) \ & \text{and } \Psi(x \leq 0) \\
\text{for } x \in \Gamma_{tip} \ (x = 0) \ & \text{and } \Psi(x = 0) 
\end{cases}$$

Where $\Gamma_{tip}$ indicates the fracture tips location.
3.3.6 Implementation of XFEM in Abaqus.

In ABAQUS when the crack propagation is simulated using XFEM, the near tip asymptotic singularity (the third term in the equation above) is not needed, and only the displacement jump across a cracked element (the second term in equation above) is considered. Therefore, the crack-tip has to propagate across an entire element at a time to avoid the need to model the stress singularity [3.35]. Level set method in ABAQUS is a numerical technique for describing a crack and tracking the motion of the crack. It couples naturally with XFEM and makes possible the modeling of 3D arbitrary crack growth without re-meshing [3.34]. Phantom nodes, which are superimposed on the original real nodes, are used to represent the discontinuity of the cracked elements. The phantom node is completely constrained to its corresponding real node when the element is intact; while the phantom node splits from the real node when the element is cut through by a crack. [3.35]

XFEM in ABAQUS makes crack modeling easy and accurate and allows cracks to be modeled independent of the mesh. Allows simulation of initiation and propagation of discrete crack along an arbitrary, solution-dependent path without requirement of re-meshing and it supports contour integral evaluation for stationary cracks [3.31].

The fracture surfaces and the fracture tip location in Abaqus are identified with a numerical procedure based on Level set Method. Once the mesh discretization has been created, each node of the finite element grid is characterized with its three coordinates with respect to the global coordinate system and two additional parameters, called PHILSM and PSILSM. These parameters are nonzero only for the enriched elements and they might be easily interpreted as the nodal coordinates of the enriched nodes in a coordinate system centered at the fracture tip and whose axes are, respectively, tangent and normal to the fracture surfaces at the fracture tip [54].

In ABAQUS, the two cracks states can be predictable [3.31][ 3.34][ 3.35]. These are:

1. Stationary cracks
2. Propagating cracks.

There are two distinct types of damage modeling for propagating cracks within an XFEM framework. These are:

1. Cohesive Segment Approach, and
2. Linear Elastic fracture mechanics (LEFM) Approach based on Virtual Crack Closure Technique (VCCT).

1. **Cohesive Segment Approach.**

   It can be used for brittle or ductile material fracture application. Uses traction separation laws and it follows the general framework for surface based cohesive behavior. The damage properties (Criteria) are specified as part of the bulk material definition.

   The pressure over closure relationship governs the behavior when the crack is “closed” and cohesive behavior contributes to the contact normal stress when the crack is “open”. [3.36]

   Crack initiation refers to the beginning of degradation of the cohesive response at an enriched element. The process of degradation begins when the stresses or the strains satisfy specified crack initiation criteria [3.36] [3.37] [3.38]. Crack initiation criteria in ABAQUS are available based on the stress and strain. These are:

   Maximum principal stress (MAXPS) and Maximum principal strain (MAXPE)

   Maximum nominal stress (MAXS) and Maximum nominal strain (MAXE)

2. **Virtual Crack Closure Technique (VCCT)**

   This method is more appropriate for fracture propagation problems in brittle materials. In this method, only the displacement jump function in cracked element is considered and the fracture has to propagate the entire element at once to avoid the need to model the stress singularity. The strain energy release rate at the fracture tip calculated based on the modified virtual crack closure Technique (VCCT). Using this approach fracture propagation along an arbitrary path can be simulated without the need to fracture path being known a priori.

   The modeling technique is similar to the XFEM-based cohesive segment approach. In this method also phantom nodes are introduced to represent the discontinuity of the enriched elements. The fracture criterion satisfied when the equivalent strain energy release rate exceeds the critical strain energy rate at the fracture tip in the enriched element [3.34].
Chapter Four Numerical simulations of crack growth in damage integral skin stringer panel using XFEM.

4.1 Morfeo/crack for Abaqus.

4.1.1 Introduction

Morfeo/Crack is a software product for the computation of the stress intensity factors (SIFs) along the front of three-dimensional cracks and the prediction of crack propagation under fatigue loading using the extended finite element method (XFEM). XFEM is an extension of the finite element method that allows the presence of cracks inside the elements and offers a high precision on the stress singularity at the crack front with special enriched degrees of freedom.

Morfeo/Crack for Abaqus is built upon the implementation of XFEM available in Abaqus since version 6.10. The functionality of Abaqus for SIF computation is however limited to the calculation of stationary cracks. Morfeo/Crack for Abaqus enhances Abaqus and is capable of performing crack propagation simulations in complex geometries. The method is based on calling Abaqus/Standard at each propagation step.

Between each step, it reads the Abaqus solution, recovers an richer, improved XFEM solution in a small area surrounding the crack using a tailored integration rule, accurately computes the stress intensity factors which determine the crack advance and updates the Abaqus input file with the new crack position. Moreover, Morfeo/Crack for Abaqus profits from the nice and intuitive user interface Abaqus/CAE since it is integrated in the latter as a plug-in for the definition of the initial crack position and the specific data for fatigue crack propagation. Finally, Morfeo/Crack for Abaqus offers the choice between post-processing the results in Abaqus/CAE as usual or in a freely available post-processor (gmsh), which renders the solution at the crack tip. [4.1]

4.1.2 Number of cycles and SIF calculations (ABAQUS)

In this article, software ABAQUS is used for Number of cycles and SIF calculation. The whole data input includes Part, property, load and so on. The modules of ABAQUS are described in the Figure 4.1
There are 6 main steps of creating a model in ABAQUS
1. Create 3D model (shape and dimensions).
2. Defining the materials, mechanical properties for all different zones.
3. Introducing the initial crack within the structure, including its shape and location.
4. Introduce the loading including its intensity, type and location within the structure.
5. Defining the boundary conditions,
6. Generating the final mesh, refined around the initial crack and in the regions were the crack expected to grow.

The x-FEM that provided by Morfeo was used for numerical simulation of fatigue crack growth.

4.2 Numerical model

4.2.1 Model I (base metal)

The main idea of numerical modeling was to test XFEM. This is done by making FE model of base metal plate with initial crack (as shown in Figure 4.2). The real loads from experiment data were used in the simulation. The number of cycles obtained numerically (obtained by the simulation) were compared with the number of cycles obtained experimentally. Base metal plate
was chosen because it had simple geometry and the calculated values of SIF could be verified using other methods or can be even found in the literature.

In this simulation aluminum alloy AA6156 T6 was used (Young’s modulus $E = 71000$ MPa, Poisson’s ratio $\nu = 0.33$).

The loads used in simulation were equal to average values of maximum tensile forces over time measured in experiments (obtained due to the courtesy of researchers from GKSS Research Center, Geesthacht). For base metal plate average maximum force was $F_{\text{max}} = 112.954$ KN, while the load ratio $R = 0.146$ was determined on the basis of average minimum tensile force measured. Coefficients for Paris equations were adopted on the basis of the values obtained in tests with base metal plates: $m = 3.174$ and $C = 1.77195 \times 10^{-12}$ MPa mm$^{1/2}$.

Initial crack in the 1st simulation was propagated to length $2a = 275$ mm, and Figure 4.3 shows its shape after the last growth step.
4.2.2 Model II (4-stringer with 1mm size of mesh). AA6156 T6

After successful numerical simulation of crack growth on base metal plate, the second has been performed on more complex geometry of 4-stringer plate as shown in (Figure 4.4). The central crack of the length $a_0=14$ mm was initiated and the load identical to that used for base metal plate was applied. The crack was propagated in the total of 173 steps (in each step crack length increased by 2 mm) and after 68 steps, it reached the wall of the left stringer (Figure 4.5) and began to spread along it. At the same time crack continued to spread through the base metal plate, reaching the wall of the right stringer after 78th step (Figure 4.6) and beginning to spread along that stringer (Figure 4.7).
Figure 4.4 Model of 4-stringer plate with 3D crack used in simulation

Figure 4.5 Crack in 4-stringer plate after 68 steps of propagation
Figure 4.6 After 78 steps crack begins to spread along the stringer.

Figure 4.7 Crack after 130 steps of propagation: both stringers are highly damaged.

During the 160th step complete failure of the left stringer occurred (Figure 4.8), after which the crack continued to spread along the right stringer and through the base metal plate. Simulation of
the crack growth stopped after 173 steps because the number of load cycles necessary to propagate
the crack by one millimeter dropped under 100, which was the sign that crack started to propagate
rapidly and that 4-stringer plate is under complete failure.

![Figure 4.8 Crack after 160 steps of propagation.](image)

### 4.2.3 Model III (4-stringer with 1mm size of mesh). AA6156 T4

In this simulation aluminum alloy AA6156 T4 were used (Young’s modulus $E = 71000$ MPa,
Poisson’s ratio $\nu = 0.33$) just we had changed Coefficients for Paris equations were adopted on the
basis of the values obtained in tests with base metal plates: $m = 3.042$ and $C=4.7.10^{-11}$ MPa mm$^{1/2}$. The central crack of the length $a_0=14$ mm was initiated and the load identical to that used for the
same pervious model was applied. The crack was propagated in the 48 steps (in each step crack
length increased by 2 mm), as shown in figure 4.9
4.3 Effect of mesh size on fatigue crack propagation behavior for 4-stringers AA6156 T6.

In these models, we changed size of mesh for previous model II (4-stringer 1mm) around crack to 2mm and 4mm,

4.3.1 Model I (4-stringer with 2mm size of mesh).

In this model the central crack of the length $a_0=14$ mm was initiated and the load identical to that used for 4-stringes was applied and changed size of mesh. The crack was propagated in the total of 117 steps (in each step crack length increased by 2 mm, 1mm left and 1mm right) and after 60 steps, it reached the wall of the left stringer (Figure 4.10) and began to spread along it. At the same time crack continued to spread through the base metal plate, reaching the wall of the left stringer after 80 step (Figure 4.11) and beginning to spread along that stringer.

Figure 4.9 Crack after 48 steps of propagation for 4-stringers AA6156 T4
Figure 4.10 Crack in 4-stringer plate with 2mm size of mesh after 60 steps of propagation.

Figure 4.11 After 80 steps crack begins to spread along the stringer.

Also after 93 steps of propagation for both stringers, we noticed that in figure (4.12) and (4.13) for left stringer is highly damaged compared to right stringer is began to damage.
Figure 4.12 Crack after 93 steps of propagation: right stringer began to damage.

Figure 4.13 Crack after 93 steps of propagation: left stringer is highly damaged.

After 100-step complete failure of the left stringer occurred Figure 4.14, after which the crack continued to spread along the right stringer and through the base metal plate. Simulation of the crack growth stopped after 117 steps in figure 4.15 4-stringer plate is under complete failure.
4.3.2 Model II (4-stringer with 4mm size of mesh).

The crack propagated in the total of 279 steps. After 76 steps, it reached the wall of the left and right stringers as shown in (Figure 4.16) and began to spread both of them. At the same time crack
continued to spread through the base metal plate. After 88 steps, the crack began to spread along the left and right stringers as shown in (Figure 4.17).

![Figure 4.16 Crack in 4-stringer plate with 4mm size of mesh after 76 steps of propagation.](image)

![Figure 4.17 After 88 steps crack begins to spread along the stringer.](image)

After 166-step, both stringers left and right were completed failure and destroyed. (First and second stringers in model) as shown in Figure 4.18, after that, the crack continued to spread and reached the third stringer.
Figure 4.18 After 166 steps completed failure and destroyed first and second stringers.

After 212-steps, third stringer began to damage. After that, the crack continued to spread along the base metal toward the third and fourth stringers as shown Figure 4.19. The Simulation of the crack growth stopped after 278 steps as shown in Figure 4.20. The 4-stringer plate with 4mm size of mesh was completed failure.
Figure 4.19 After 212 steps third stringer began to damage.

Figure 4.20 After 278 steps third and fourth stringers began to damages.

4.3.3 Model III (4-stringer with 2mm size of mesh and (toe)).

This model was same as the previous models with a small changing. The materials and the dimensions were remains same. This model has a (LBW) welding. The dimension of the LBW
welding was 2mm for both the sides. The initial crack length was 14 mm in a symmetrical position. The load ratio was $R=0.146$.

Figure 4.21 shows LBW welded stiffened panels. Figure 4.22 shown the Model III of 4-stringer plate.

![LBW welded stiffened panels diagram](image)

**Figure 4.21** LBW welded stiffened panels
Figure 4.22 Model of 4-stringer plate with (toe).

The crack propagated in the total of 139 steps. After 30 steps, it reached the wall of the left and right stringers as shown in (Figure 4.23) and began to spread both of them. At the same time crack continued to spread through the base metal plate. After 88 steps, the crack began to spread along first and second stringer as shown in figure (4.24).

Figure 4.23 Crack in 4-stringer plate with 2mm size of mesh with toe after 30 steps of propagation.
Figure 4.24 After 88 steps completed failure first and second stringers.

Figure 4.25 After 103 steps crack propagated toward third and fourth stringers.
After 103-steps, third stringer began to damage, and continued to spread along the base metal toward the fourth stringer as shown Figure 4.25. The Simulation of the crack growth stopped after 138 steps as shown in Figure 4.26. The 4-stringer plate with 2mm size of mesh with (toe) was completed failure.

4.4. XFEM ABAQUS RESULTS.

The crack growth results data we got it by XEFM for different sizes of mesh for 4-stringer panel and base metal configurations was done by Excel considering the variation of the Number of cycles (N) VS Crack length (a_mm).

Figure 4.26 Crack after 138 steps of propagation.

Figure 4.27 Crack propagation vs. cycle number N for base metal (XEFM).
Figure 4.28 Crack propagation vs. cycle number N 4-stringer (1mm) (XEFM).

Figure 4.29 Crack propagation vs. cycle number N, 4-stringer (1mm) T4 (XEFM).
Figure 4.30 Crack propagation vs. cycle number N 4-stringer (2mm) (XEFM).

Figure 4.31 Crack propagation vs. cycle number N 4-stringer (4mm) (XEFM).
Figure 4.32 Crack propagation vs. cycle number $N$ for 4-stringer (2mm) with toe (XFEM)

Figure 4.33 Effect of mesh size on fatigue crack propagation behavior for 4-stringers.
All simulation analyzes are performed using ABAQUS/Morfeo software, the previous figures show that relation between the number of cycles VS crack length. ALL models in which the effected of the size of mesh give better results than the model with 1mm size of mesh as shown in figure (4.33). The results were obtained for fatigue life of the cracked structural for models are conservative. This is good in practical design and analysis with respect to fracture mechanics and life estimations.

In Figure (4.34), it can be noticed that the fatigue life for A6165 T4 is higher than A6165 T6 that’s mean how Coefficients for Paris equations C and m are effected on fatigue life.
4.5 4-stringer with 3-clips 2mm size of mesh.

The geometry of the 3-clips structure shown in (Figure 4.35), was modelled after numerical simulation of 4-stringer plate with different size of meshes, in this part we also had performed much geometry for 3-clips with different meshes but we had mentioned only one model 3-clips with 2mm size of mesh. The central crack of the length $a_0=17$ mm was initiated and the load identical to that was used for 4-stringer plate applied. The crack was propagated in the total of 91 steps (in each step crack length increased by 2 mm) and after 14 steps, as shown in (Figure 4.36) first clip began to deformed along it. At the same time, crack continued to spread through the base metal plate, reaching the wall of the right, left stringers after 91 steps as shown in (Figure 4.37), and beginning to spread along those stringers.

![Figure 4.35 geometry of 4-stringer plate with 3-clips](image-url)
Figure 4.36 Crack after 14 steps of propagation for 4-stringers with 3-clips

Figure 4.37 Crack after 91 steps of propagation: left, right stringers are damaged and 1-clip deformed.
It can be noted that in figure (4.38) after modelled 4-stringer plate with 3-clips, XFEM simulation number of cycles for 3-clips is higher than 4-stringer, (278476.44 cycles versus 264958.27) cycles, which is a difference of about 13518.17 cycles (4.85%). This difference in Number of cycles, which can lead us to enhance fatigue life.
Chapter 5 Experimental Validation of Numerical Results (XFEM).

5.1 Introduction.

This chapter will start by introducing the material compositions as well as the literature experimental data for the base metal and four-stringer panel integral structure. All experimental results are here presented as a reference from Bremen and GKSS research Centre. It will also describe the techniques that were applied to analyze the data obtained from the experimental procedures. The experimental procedure and the results obtained are presented and discussed. This chapter is concerned only with experimental data while the modelling work was described in the previous Chapter.

5.1.1 Materials and its properties.

AA6156 is an improved AA6056 endowed with an enhanced damage tolerance similar to the one showed by alloys of the 2xxx series, due to impurity reduction and narrower allowed composition range of alloying elements. In T4 temper, AA6156 presents a good formability while aged to T6 develops an improved toughness and a high resistance to fatigue crack growth. To assure high corrosion resistance, especially to the intergranular corrosion at high temperature [5.1]

<table>
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Table 5.1 chemical composition (wt-%) of AA6156.

5.1.2 Literature experimental data.

The reference of experimental data is taken from Bremen and GKSS research center. This report presented on “European Workshop on Short Distance WELding Concepts for AIRframes - WEL-
AIR” on June 2007 that is created on the basis of damage tolerance analysis of 4-stringer flat panels that are jointly made by the Airbus division in Bremen and GKSS Research Center Geesthacht (Hamburg) – Germany. By courtesy of project participants, the results of fatigue test of laser beam welded short distance clip welds using 4-stringer flat panels [5.2] were available for inspection and they were used as reference for verification of fatigue life values obtained by numerical simulations using XFEM.

The main idea of the project was to perform fatigue testing of integral structures that should replace the conventional differential structures (Figure 5.1) where joints are obtained using rivets. Panels with stringers are traditionally used in fuselage production; therefore, Airbus has decided to test this type of geometry.

Figure 5.1 Differential vs. integral structure of the fuselage
5.2 AA6156 T6 base Metal.

The investigated for base metal was configurations with a thin sheet. The geometry and the dimensions of the base metal given in (Figures 5.2) sizes of 760 mm × 1200 mm and 2.6 mm thickness. For base metal plate average maximum force was $F_{\text{max}}=112.954$ KN, while the load ratio $R=0.146$ was determined on the basis of average minimum tensile force measured. Coefficients for Paris equations were adopted on the basis of the values obtained in tests with base metal plates (Figure 5.3): $m = 3.174$ and $C=1.77195E-012$ MPa mm$^{1/2}$.

![Figure (5.2) Geometry of the base metal.](image)

![Figure 5.3 Determination of Paris coefficients on base metal plate](image)
### Table 5.3 X-FEM and Experimental data for base metal.

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<th>Stress intensity factor, KI MPa√mm</th>
<th>Number of cycles</th>
<th>Crack growth rate, da/dN (m/cycle)</th>
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### 5.2.1 Results and discussion
As it can be seen in Figure 5.4, the number of cycles predicted by Paris equation incorporated into Morfeo/Crack for ABAQUS software is comparable to the number of cycles obtained in one of the experiments with base metal plate (different values of number of cycles were obtained in series of experiments; however, the deviation was not greater than 15%).

Figure 5.4. Numbers of cycles obtained in experiment and XFEM simulation (base metal T6).

Figure 5.4 shows that in XFEM simulation number of cycles to critical crack length is less than that obtained in experiment (169076 cycles versus 189514 cycles, which is a difference of about 10%); however, under crack length 2a=60mm (almost linear growth) the numbers of cycles differ insignificantly. This was also confirmed by comparing SIFs values obtained by XFEM and by software NASGRO [5.3] (Figure 5.5). It is evident that.

Morfeco/Crack for Abaqus calculates higher Mode I SIFs compared to NASGRO and due to that fact the predicted fatigue life is shorter; however, the number of cycles to critical crack length is on the safe side – predicted life is shorter than that obtained in experiment. (It is important to
mention that NASGRO calculates SIFs at the tip of the 2D crack, while Morfeo/Crack for Abaqus calculates SIFs at the nodes of 3D crack front.

Figure 5.5 SIF values obtained in NASGRO software and XFEM simulation (base metal T6)

5.3 AA6156 T6 Four-stringer panel

Integral skin-stringer structure is obtained using laser beam welding (LBW), and fatigue life testing was performed on panel under tension containing growing damage perpendicular to the stringer weld joint (circumferential crack, as shown in Figure 5.6). Panel geometry with stringers and their dimensions are presented in (Figure 5.7).
4-stringer panels were tested by a testing machine shown in Figure 5.8. The initial crack length was 14 mm. The values of maximum applied tensile force (with constant amplitude and stress ratio) were varied. In tests, different aluminum alloys (6156T6, 2139T8, 6156T4) were used. The fatigue characteristics, (represented by Paris coefficients) were determined by testing the base metal panels without stringers (Figure 5.3). Base metal panels dimensions, tension force and initial
crack lengths were identical to dimensions of 4-stringer panel. The fatigue crack lengths during testing were measured using remote optical microscope.

Figure 5.8 Equipment used in fatigue testing.

5.3.1 Results and discussion

Model I (4-stringer with 1mm size of mesh).

Figure 5.9. Numbers of cycles obtained in experiment and XFEM simulation (4-stringer 1mm).
Figure 5.9 shows that in XFEM simulation number of cycles to critical crack length is less than that obtained in experiment (254273.7723 cycles versus 422328 cycles, which is a difference of about 40%).

Having in mind that the researchers from GKSS Research Center have also investigated 4-stringer plates with crack, but the results were not completely accessible; in order to validate the results of simulation we compared the values of crack growth rate obtained by XFEM to available experimental crack growth rate values (Figure 5.10). As it can be seen in (Figure 5.11), the numerical values are very close to experimental ones.

Figure 5.10  Comparison of the crack growth rate for the base metal plate (black dots) and 4-Stringer plate (blue dots) obtained in the experiment.
Figure 5.11 Comparison of the crack growth rate for the base metal plate obtained in the experiment and 4-stringer plate obtained in simulation with XFEM Model II (4-stringer with 2mm size of mesh).

Figure 5.12. Numbers of cycles obtained in experiment and XFEM simulation (4-stringer 2mm).
Figure 5.12 shows the number of cycles to critical crack length (XFEM) is less than from number of cycles obtained in experiment (273230.187 cycles versus 422328 cycles, which is a difference of about 35%).

Figure 5.13 Comparison of the crack growth rate for 4-stringer obtained in the Experiment and 4-stringer plate (2mm) obtained in simulation with XFEM Model III (4-stringer with 4mm size of mesh).

Figure 5.14. Numbers of cycles obtained in experiment and XFEM simulation (4-stringer 4mm).
As Figure 5.14 shows, the number of cycles to critical crack length (XFEM) is still less than from number of cycles obtained in experiment (290743.6058 cycles versus 422328 cycles, which is a difference of about 31%).

Figure 5.15 Comparison of the crack growth rate for 4-stringer obtained in the Experiment and 4-stringer plate (4mm) obtained in simulation with XFEM.

IV (4-stringer with 2mm size of mesh and (toe)).

Figure 5.16. Numbers of cycles obtained in experiment and XFEM simulation (4-stringer 4mm).
As Figure 5.16 shows the number of cycles to critical crack length (XFEM) is still less than from number of cycles obtained in experiment (290743.6058 cycles versus 422328 cycles, which is a difference of about 31 %).

Figure 5.17 Comparison of the crack growth rate for 4-stringer obtained in the Experiment and 4-stringer plate (2mm with toe) obtained in simulation with XFEM.
REFERENCES


[1.6]. E. Seib, Residual strength analysis of laser beam and friction stir welded aluminium panels for


[3.6]. http://www.efunda.com


[3.27]. Awas Ahmed, Extended Finite Element Method (XFEM)-Modeling arbitrary discontinuous and failure analysis, MSc thesis, University of studi da Pavia, April, 2009.

[3.28]. DibakarDatta, Introduction to Extended Finite Element (XFEM) Method, Erasmus MSc in computational mechanics (No: 080579k), France.


[3.36]. Adam Bevan et al, Development and validation of a wheel wear and rolling contact fatigue damage model, University of Huddersfield, Institute of Railway Research, 2013, UK.
[3.37]. J.J Kalker; Wheel-Rail Rolling Contact theory; Wear, 144(1991) 243-261; Netherlands.


