INITIAL PLASTIC DEFORMATION AND RESIDUAL STRESS INFLUENCING THE WELDING JOINT BEHAVIOR IN THE PRESENCE OF CRACKS

Doctoral Dissertation

Belgrade, 2015
Dedicated to my wife, parents and teachers
MENTOR:

DR. Aleksander Sedmak, full professor

University of Belgrade, Faculty of Mechanical Engineering

MEMBERS OF COMMISSION:
Acknowledgement

I would like to express my special appreciation and thanks to my advisor professor Dr. Aleksander Sedmak, you have been a tremendous mentor for me, I would like to thank you for encouraging my research and for allowing me to grow as a research scientist. Your advice on both research as well as on my career have been priceless.

A special thank is given to the staffs of Mechanical Engineering faculty. I appreciate their help and their kindness.
Abstract

The full-scale model of penstock was produced using weldable high strength low alloyed steel (HSLA) Sumiten 80P (SM 80P). Steel SM 80P belongs to HT80 steel with tensile strength above 800 MPa and yield strength above 700MPa. Tensile properties were achieved by quenching and tempered technology which requires strong obeying of welding procedures. Finite element analysis has been carried out by using ABAQUS software to simulate the hydrostatic test of the full-scale model of penstock. A finite element model of penstock is a little bit different than the experimental test model in welding the shape and geometry (the third segment of the cylindrical mental of the experimental model has been neglected for the finite element model). In the first portion of the analysis the von Misses stress distribution will be investigated in two steps, the first load-unload and the second load-unload, and to focus on where the yielding initiates and spreads. For the second portion of the numerical study, the behavior of the model with initial residual stresses in weld joints have been analyzed for von Misses stresses distribution. The von Misses stress-strain relationship has been calculated in three ways: using linear elastic formulas, when the experimental model was treated as an ideal cylindrical vessel (without a 5° angle). The second relationship is obtained from strain guage measurements, and the third behavior is the stress-strain curve obtained from a numerical calculation (ABAQUS software). For the residual strength prediction and structural integrity assessment of penstock, a study of fracture mechanics behavior of an under-matched weld joints with small and large surface cracks for high strength low alloy steel of penstock structures has been performed by the J-R curve approach. Suminet 80P (SM 80P) grade steel plate was butt welded by submerged arc welding. Three tensile panels with surface cracks positioned in the base metal (BM), weld metal (WM) and the heat affected zone (HAZ) were tested at room temperature. And continuous measurement of force versus crack mouth opening displacement and crack extension was monitored during the test by the compliance method. In addition, J-R curves were built for three parts of the weld joint. Crack driving force is obtained for various values of applied stresses ratio and it plotted as a function of crack depth ratio.
Извод

Модел цевовода пуне размере је произведен од заварљивог нисколегираног челика високе чврстоће Sumiten 80P (SM 80P). Челик SM 80P припада групи HT80 челика којима је заварена чврстоћа изнад 800 MPa и напон течења изнад 700 MPa. Затезне карактеристике су последица каљења и отпуштања, стога се приликом процеса заваривања мора испоштовати одговарајућа процедура. Уз помоћ софта ABAQUS, методом конечних елемената на моделу је извршена симулација хидростатичког теста. Модел који је коришћен за прорачун методом конечних елемената разликује се од експерименталног модела у изгледу завареног шава и геметрије шава (трећи сегмент цилиндричног експерименталног модела је приликом прорачуна методом конечних елемената записан). У првом делу анализе промена вон Мизесових напона ће се испитати у два корака, први корак оптерећења-растерећења и други корак оптерећења-растерећења и фокус ће бити позиције почетка течења материјала и његова ширење. У другом делу нумеричког прорачуна анализирани је понашање модела у коме су генерисани иницијални заостали напони. Релација вон Мизесових напона и деформација је рачуната на три начина: линеарно-еластична анализа, када се модел третира као идеално цилиндрична посуда (без угла од 5º). Друга релација је добијена уз помоћ експерименталног мерења деформације и треће, понашање напон-деформација криве добијене нумеричким прорачуном (ABAQUS софтвер). За претпоставке процене преостalog века и процене интегритета конструкције, коришћена је студија механике лома испитивањем J-R криве метала шава ниже чврстоће са малом и великим прслином за нисколегиране челике високе чврстоће. ЕПП поступак је коришћен за чеоно заваривањеSuminet 80P (SM 80P )плоче. Контруално мерење силе и уста отварања као и ширење прслине је посматрано током теста одговарајућим методама. Поред тога, J-R криве су добијене за три различите позиције завареног споја. Сила раста прслине је добијена за различите вредности употребљеног напона и графички је представљен у функцији дубине прслине.
## CONTENTS

### Chapter 1: Introduction

1-1. Introduction to the pressure vessel .................................................................1  
  1-1-2. Material of construction of pressure vessel .............................................2  
  1-1-3. The mechanical properties that generally are of interest ..........................2  
  1-1-4. Cylindrical pressure vessel ..................................................................3  
  1-1-5. Hoop (Circumferential) stress ................................................................3  
  1-1-6. Axial (Longitudinal) stress ..................................................................4  

1-2. Introduction to fracture mechanics .............................................................5  
  1-2-1. Linear elastic fracture mechanics ..........................................................6  
  1-2-2. Elastic-plastic fracture mechanics .........................................................14

### Chapter 3: Finite Element Analysis of Solids

2-1. Introduction .................................................................................................23  
2-2. Formulation of finite element equation ......................................................23  
2-3. Linear-elastic finite element analysis ..........................................................23  
  2-3-1. Three dimensional isoparametric elements ...........................................25  
2-4. Elastic –plastic finite element analysis .......................................................30  
  2-4-1. The mathematical theory of plasticity .................................................31  
  2-4-2. The yield criteria ...............................................................................31  
  2-4-3. The Von Mises criterion .....................................................................32  
  2-4-4. Work or strain hardening ...................................................................34  
  2-4-5. Elastic-plastic stress/strain relation .....................................................35  
2-3-6. Matrix formulation ...............................................................................36
Chapter 3: Structural Integrity Assessment of Pressure Vessel

3-1 Introduction

3-2. Failure Assessment Diagram approach (FAD)

3-3. R-Curve approach

3-3-1. Specimen configuration

3-3-2. $K_{IC}$ testing

3-3-3. K-R Curve testing

3-3-3-1. Experimental measurements of K-R curves

3-3-4. $J_{IC}$ testing

3-3-4-1. J-R Curve

Chapter 4: Methodology and approach of FEA and experimental work

4-1. Introduction

4-2. Methodology and approach of FEA

4-2-1. Created the geometry of model in ABAQUS/CEA

4-2-2. Mechanical properties of FE model

4-2-3. Mesh of FE model

4-2-4. Boundary condition and loading

4-2-5. Initial residual stresses for first load

4-2-6. Initial residual stresses for second load

4-3. Experimental work of hydrostatic test of a full-scale model of penstock

4-3-1. Introduction

4-3-2. Hydrostatic testing of the model
Chapter 5: Results and discussion

5-1. Finite element analysis (ABAQUS) of full-scale model of penstock

5-1-1. Von Misses stresses distribution of FE model for FL

5-1-2. Plastic strain (FL-UNL)

5-1-3. Von Misses stress distribution of FE model for SL

5-1-4. Plastic strain (SL-UNL)

5-1-5. Von Misses stress-strain curve of weld joint LSI SAW without RS

5-1-6. Von Misses stress-inner Pressure curve of weld joint LSI SAW without RS

5-1-7. Inner Pressure-Von Misses strain curve of weld joint LSI SAW without RS

5-1-8. Hoop stresses-strains curves of weld joint LS1 SAW without RS

5-1-9. Von Misses stresses distribution of FE model for FL (with RS)

5-1-10. Plastic strain (FL-UNL, with RS)

5-1-11. Von Misses stresses distribution of FE model for SL (with RS)

5-1-12. Plastic strain (SL-UNL, with RS)

5-1-13. von Misses stresses-strains curve of weld metal LS1 SEW
5-1-14. Hoop stresses- strains curve of weld metal LS1 SEW

5-2. Experimental results with numerical calculations of full-scale model of penstock

5-3. Residual strength prediction of penstock by using R-curve

5-3-1. Stresses-CMOD relationship

5-3-2. J-R curves of small and large surface flaw

5-3-3. Results of failure prediction of base metal, weld metal and heat affected zone

Conclusion
LIST OF FIGURES

Figure 1-1, Horizontal cylindrical pressure vessel in steel ............................................. 1
Figure 1-2, biaxial state of stresses, hoop stresses and axial stresses .............................. 3
Figure 1-3, determination of hoop stress, at the diametrical cut .................................... 3
Figure 1-4, determination of longitudinal stress at vertical cut ...................................... 4
Figure 1-5, elliptical hole in flat plate ............................................................................. 6
Figure 1-6, stress field ahead of crack tip for mode I ...................................................... 7
Figure 1-7, unloaded area around free surface ................................................................ 8
Figure 1-8, the fracture energy balance ......................................................................... 9
Figure 1-9, three fracture modes .................................................................................. 10
Figure (1-10), the first order and second order estimates of plastic zone (r_y, r_p) ............ 11
Figure 1-11, the strip yield model ................................................................................ 12
Figure 1-12, alternative definition of CTOD .................................................................. 14
Figure 1-13, CTOD in the Irwin plastic zone correction ............................................... 15
Figure 1-14, Estimation of CTOD from strip yield model ............................................. 16
Figure 1-15, superposition of two load cases for the Dugdal model ............................... 16
Figure 1-16, definition of CTOA ................................................................................ 17
Figure 1-17, stress-strain behavior of elastic-plastic and non-linear elastic material ..... 18
Figure 1-18, arbitrary contour around the tip of crack .................................................. 19
Figure 1-19, stress-strain curve ................................................................................... 20
Figure 1-20, integration area ....................................................................................... 21
Figure 1-21, the dimensionless constant in for plane stress and plane strain ............... 22
Figure 2-1, linear - quadratic elements and their representation in local coordinate ....... 25
Figure 2-2, Von Mises and Tresca yield surface in principle stress coordinate………33
Figure 2-4, traditional Newton-Raphson method vs. arc-length method…………………………..39
Figure 3-1, failure assessment diagram FAD…………………………………………………………..41
Figure 3-2-a, 3-2-b, standardized fracture mechanics test specimen …………………..42
Figure 3-2-c, 3-2-d, 3-2-e, standardized fracture mechanics test specimen ……………..43
Figure 3-3, three types of load- displacement behavior in $K_{IC}$ test……………………………….44
Figure 3-4, the effect of specimen thickness on the fracture toughness in titanium………….45
Figure 3-5, the effect of ligament length on fracture toughness in aluminum alloy ………..46
Figure 3-6, K-R curve ……………………………………………………………………………………..47
Figure 3-7, load-displacement curve of crack growth in the absence of plasticity……….48
Figure 3-8, load-displacement behavior of crack growth with plasticity………………...48
Figure 3-9, side groove in a fracture mechanics test specimen……………………………………..50
Figure 3-10, plastic energy absorbed by specimen test……………………………………………….51
Figure 3-11, compliance method for J-R curve…………………………………………………………..51
Figure 3-12, J-R curve for A 710 Steel………………………………………………………………….52
Figure 4-1, finite element model of penstock as sketched in ABAQUS/CEA…………………53
Figure 4-2, mesh of FE model…………………………………………………………………………….54
Figure 4-3, Boundary condition and applied inner pressure…………………………………………..55
Figure 4-4, initial residual stresses for FL of FE model……………………………………………….56
Figure 5-5, initial residual stresses for SL of FE model…………………………………………….56
Figure 4-6, Design of penstock segment full-scale model……………………………………..58
Figure 4-7, Instrumentation and specimens sampling in penstock model…………………...60
Figure 4-8, surface flaw geometry……………………………………………………………………….65
Figure 4-6, preparation samples for tensile panel test…………………………………...66
Figure 4-7, CDF of base metal of penstock

Figure 5-1, von Misses stresses distribution of FE model of first load, (P =14.5MPa)

Figure 5-2, plastic deformation of FE model (FL-UNL, P=14.5MPa)

Figure 5-3, von Misses stresses distribution of FE model of second load, (P=18.5MPa)

Figure (5-4), plastic deformation of FE model (SL-UNL, P=18.5MPa)

Figure 5-5, Von Misses stress-strain behavior LS1 SAW without RS

Figure 5-6, Von Misses-Inner Pressure behavior of WM LS1 SAW

Figure 5-7, inner Pressure-Von Misses strain of LS1 SAW

Figure 5-8, Hoop stresses-strain curve of WM LS1 SAW

Figure 5-9, von Misses stresses distribution of FE model for first load with RS

Figure 5-10, plastic strain of WM LS1 SAW after FL

Figure 5-11, Von Misses distribution of FE model for SL with RS

Figure 5-12, plastic strain of WM LS1 SAW after SL

Figure 5-13, von Misses stresses-strain curve of WM LS1 SAW with RS

Figure 5-14, Hoop stresses- strain of WM LS1 SAW with RS

Figure 5-15, Relationships Stress – Strain for ideal cylinder

Figure 5-16, BM Von Misses Stress vs. Inner Pressure comparison with FE calculations

Figure 3-17, Von Misses Stress- Strain relationships of BM for different calculation

Figure 5-18, Relationships between inner pressure and von Misses strain

Figure 5-19, Stress – Strain distribution for penstock model

Figure 5-20, Von Misses Stress – Inner Pressure relationship

Figure 5-21, Von Misses Stress – Strain relationships for SMAW weld joint LS1

Figure 5-22, Von Misses Stress – Strain relationships for SMAW weld joint LS2

Figure 5-23, Comparison of Von Misses stress strain relationships for joints LS1 and LS2
Figure 5-24, Hoop Strain against applied inner pressure during both loading sequences.92
Figure 5-25, Hoop Stress – Strain distribution for LS3 MAW joint .................................92
Figure 5-26, Von Misses Stress – Strain distribution for L3 SMAW joint ..............................93
Figure 5-27, Inner pressure – Hoop strain for L4 MAW ....................................................94
Figure 5-28, Hoop Stress – Hoop strain for L4 MAW .......................................................94
Figure 5-29, Von Misses Stress – Strain for L4 MAW .......................................................95
Figure 5-30, Comparison between the welded joints stress–strain distribution in the second cylinder ..................................................................................................................95
Figure 5-31, Inner pressure vs. Hoop strains for CM MAW circular weld joint .................96
Figure 5-32, Hoop Stress vs. Hoop strains for CM MAW circular weld ...............................96
Figure 5-33, Von Misses Stress vs. Hoop strains for CM MAW circular weld together with FE calculations ..................................................................................................................97
Figure 5-34, Comparison of Von Misses Stress – Strain relationships for two circular weld joints ...........................................................................................................................97
Figure 5-35, stresses-CMOD relationship of SSF ................................................................98
Figure 5-36, stresses-CMOD relationship of LSF ................................................................98
Figure 5-37, J-R curves of WM, BM and HAZ (LSF, SSF) .....................................................99
Figure 5-38, J-R curves for WM, BM, HAZ (SSF) .................................................................100
Figure 5-39, determining the point of instability of BM of penstock ...............................101
Figure 5-40, determination the point of instability of WM of penstock ............................102
Figure 5-41, determination the point of instability of HAZ of penstock ............................103
1-Introduction

1-1. Introduction to the pressure vessel

Vessels, tanks, and pipelines that carry, store or receive fluids are called pressure vessels. A pressure vessel is defined as a container with a pressure differential between the inside and the outside. The pressure vessel often has a combination of a high pressure together with high temperature, and in some cases flammable fluid or highly radioactive materials. The size and geometric form of pressure vessels vary greatly from large cylindrical vessels used for high-pressure gas storage to the small sized ones used as hydraulic units for aircrafts.

It is important for a pressure vessel designer to understand the nature of loading that acting on the pressure vessel and the structural response to the loading. Generally the loads acting on a structure can be classified as:

- Sustained
- Deformation controlled
- Thermal

Figure (1-1), Horizontal cylindrical pressure vessel in steel

These three loads types may be applied in a steady or cyclic manner. The structure under the action of these loads may respond in a number of ways [1]:
- When the response is elastic, the structure is safe from collapse when the applied loading is steady. If the applied loading is cyclically a failure due to fatigue is expected, (high cycle fatigue)

-When the response is elastic in some regions and plastic in others, produced by sustained and deformation controlled loads, there is potential to have an unacceptably large deformation.

- Cyclic loads or cyclic temperature distribution can produce plastic deformation and cause fatigue failure (low cycle fatigue).

- Sustained loads in brittle materials or in ductile materials at low temperatures could result in fatigue failure (low cycle fatigue).

The failure that pressure vessel are to be designed against are generally stress dependent. For this reason it becomes necessary to obtain the stress distribution in pressure vessel.

1-1-2. Material of construction of pressure vessel:
- Steels
- Non-ferrous materials such as aluminum and copper.
- Specially metals such as titanium and zirconium
- Non-metallic materials such as plastic composites and concrete.
- Metallic and non-metallic protective coating.

1-1-3. The mechanical properties that generally are of interest are:
- Yield strength.
- Ultimate strength.
- Reduction of area.
- Fracture toughness.
- Resistance to corrosion.

Two common pressure vessel geometries are cylindrical and spherical. The thickness of a vessel wall is often small compared to its diameter, the outward pressure of the contained gas or liquid is resisted by tensile strength in the walls of the pressure vessel.
1-1-4. Cylindrical pressure vessel

A thin-walled cylindrical vessel has outer radius $R$, wall thickness $t$, and contains pressure $P$. The walls of pressure vessel are subjected to a biaxial state of stresses figure (1-2),

![Biaxial state of stresses](image)

Figure (1-2), biaxial state of stresses, (hoop stresses and axial stresses).

1-1-5. Hoop (Circumferential) stress:

The hoop stress $\sigma_h$ is caused by the pressure acting to expand the circumference of the vessel. The hoop stress is calculated by taking a horizontal cut through the diametrical plane figure (1-3). The pressure force is counteracted by hoop stress in pressure vessel wall. The corresponding force $F_W$ in the walls:

![Determination of hoop stress](image)

Figure (1-3), determination of hoop stress, at the diametrical cut
\[ F_W = \sigma_H [2tL] \]  
(1-1)

Equating the two forces to satisfy equilibrium:
\[ P[2R - \Omega L] = \sigma_H [2tL] \]  
(1-2)

\[ \sigma_H = \frac{PR}{\pi \left( 1 - \frac{t}{R} \right)} \]  
(1-3)

Since \( t \) is small compared to \( R \) the hoop stress:
\[ \sigma_H = \frac{PR}{\pi \epsilon} \]  
(1-4)

1-1-6. Axial (Longitudinal) stress:

The longitudinal stress \( \sigma_l \) is caused by pressure acting against the cylinder end caps. The longitudinal stress is calculated by considering the forces on the cross-section of the cylinder figure (4-1):

\[ p \]
Figure (1-4), determination of longitudinal stress at vertical cut.

\[ F_z = Pn(R - t)^2 = PnR^2 \left[ 1 - \frac{t}{R} \right]^2 \quad (1-5) \]

\[ PnR^2 \left[ 1 - \frac{t}{R} \right]^2 = \sigma_l 2nRt \quad (1-6) \]

\[ \sigma_l = \frac{PR}{2t} \quad (1-7) \]

Hoop stress equal two times of axial stress in cylindrical pressure vessel.

1-2. Introduction to fracture mechanics

Failure has occurred for many reasons, including uncertainties in the loading or environment, defects in the materials, inadequacies in design. Design against fracture has a technology of its own, and this is a very active area of current research.

The main modes of mechanical failure are

- Failure in elastic deformation region (buckling)
- Failure after plastic deformation (yielding and necking)
- Failure by fast fracture (cracking)

Welded structures are only occasionally exposed to buckling, that can be prevented by convenient structural geometry. And failure by plastic deformation would occur when the applied stress was exceeding to the material's yield strength. To avoid this kind of failure the engineer follows a design code which ensures that the calculable stresses in his structure will not exceed the yield strength of materials.

The third type of modes of fracture has been produced by applied stresses less than the design stresses using safety factor. The structural integrity design requires consideration of factors that determine structural performance. It includes service environment, structural function, metallurgical properties, fabrication quality, inspection requirements, quality control, and factors that are specific to weldment. All of these
factors interact with the fracture mechanics aspects because of their influence on crack size, stress and fracture properties. The knowledge and practical application of fracture mechanics are required in modern design.

The practical design use of fracture mechanics is highly dependent on experience which was evolved by structural integrity technology specialists. Engineering experience in the safe design of structures and in failure analysis is an important aspect for all practical application of fracture mechanics [2].

1-2-1. linear elastic fracture mechanics

Linear elastic fracture mechanics is the basic theory of fracture, started by Griffith (1921-1942), and completed in its essential aspects by Irwin (1957-1958) and Rice (1968). Elastic theory deals with sharp cracks in elastic bodies, and is applicable to any material as long as certain conditions are met. These conditions are related to the basic ideal situation analyzed in which all the material is elastic except in a small region (a point) at the crack tip. If the size of the plastic zone is small relative to the linear dimension of the body, the disturbance introduced by this plastic region is also small and, in the limit, LEFM is verified exactly.

1-2-1-1. Stress concentration

Definition of toughness began with the work of Inglis in 1913. [3] Inglis showed that the local stresses around a corner or hole in a stressed plate could many times higher than the average applied stress. The presence of sharp corners, notches, or cracks serves to concentrate the applied stress at these points. Inglis showed that, the degree of stress magnification at the edge of the hole in the stressed plate depended on the radius of curvature of the hole. The simplest case is defined as the Kirsch problem where different results are obtained for elliptic, square, rectangular, and other forms in a plate of finite size as well as for biaxial tension [4]. For the elliptical hole in a flat plate Fig (1-5), the stress at the tip of the major axis (A) is given by:
Figure (1-5), elliptical hole in flat plate [5].

\[ \sigma_A = \sigma \left(1 + \frac{2a}{b}\right) \]  
(1-8)

\[ \sigma_A = 2\sigma \sqrt{\frac{a}{\rho}} \]  
(1-9)

Where:

a – major axis of ellipse

b – minor axis

\( \rho \) – ellipse root radius \( \rho = \frac{b}{a} \)

If minor axis tends to zero, normal stress will tend to infinity, and in elastically deformed material the condition for fracture is fulfilled.

1-2-1-2. Stress intensity factor

George R. Irwin. [6] became interested in the fracture of steel armor plating during penetration by ammunition. His experimental work at the U.S. Naval Research Laboratory in Washington, D. C. led in 1975 to a theoretical formulation of fracture that continues to find wide application[4]. Irwin showed that the stress field \( \sigma(r, \theta) \) in the vicinity sharp crack tip could be described mathematically by:
The basic relationship for mode I crack growth between stress intensity factor ahead the crack tip \( K_I \), crack length \( a \) and applied stress \( \sigma \) is derived in term of coordinate \( (x, y, z) \) in crack surface direction:

\[
\sigma_{xx} = K_I \sqrt{2\pi r} \cos \left( \frac{\theta}{2} \right) \left[ 1 - \sin \left( \frac{\theta}{2} \right) \sin(3\theta/2) \right] \\
(1-10)
\]

Inglis’s theory showed that the stress increase at the tip of a crack of flaw depended only on the geometrical shape of the crack and not its absolute size, this seemed contrary to 1-2-1-3. Energy balance criterion. Inglis’s theory showed that the stress increase at the tip of a crack of flaw depended only on the geometrical shape of the crack and not its absolute size, this seemed contrary to
the well-known fact that large cracks are propagated more easily than smaller one. This fact led Griffith [7], to a theoretical analysis of fracture based on the point of view minimum potential energy. Griffith proposed that the reduction in strain energy due to the formation of crack must be equal or greater than increase in surface energy required by the new crack face[11].

![Diagram](image)

**Figure (1-7), unloaded area around free surface**

The strain energy per unit volume of stressed material is:

\[
U = \frac{1}{V} \int f \, dx \quad ; \quad \sigma = \frac{f}{A} \quad ; \quad U = \int \sigma \, da
\]  

(1-15)

If the material is linear, \( \varepsilon = \frac{E \sigma}{2} \) then the strain energy per unit of volume is :

\[
U = \frac{E \varepsilon^2}{2} = \frac{\sigma^2}{2E}
\]

(1-17)
When a crack has grown into a solid to a depth \( a \), a region of material adjacent to the free surface is unloaded, and its strain energy released. Using the Inglis solution, Griffith was able to compute just how much energy this is.

The total strain energy released is:

\[
U = \frac{\sigma^2}{2E} \cdot \pi a
\]

(1-18)

In forming the crack, bonds must be broken, the surface energy needed to create two surfaces is:

\[
S = 2\gamma a
\]

(1-19)

The total energy is:

\[
W = U + S
\]

(1-20)

Figure (1-8), the fracture energy balance.

The maximum in the total energy is given by:

\[
\frac{\partial W}{\partial a} = 0
\]

(1-21)
For a given crack length \( a \), the Griffith fracture stresses is given by:

\[
\frac{\sigma^2 \pi a}{E} + 2\gamma = 0
\]

(1-22)

\[
\frac{\sigma^2 \pi a}{E} = 2\gamma
\]

(1-23)

For a given crack length \( a \), the Griffith fracture stresses is given by:

\[
\sigma_f = \sqrt{\frac{2E\gamma}{\pi a}}, \text{ in plane stress}
\]

\[
\sigma_f = \sqrt{\frac{2E\gamma}{m(1-v^2)a}}, \text{ in plane strain}
\]

1-2-1-4. Fracture modes

Irwin had shown that, three modes of crack surface displacement are possible, and they describe crack behavior in all stress states, but our interest centers mainly on the type \( I \) loading, the most common type that lead to brittle failure.

![Fracture modes diagrams](image)

Figure(1-9), three fracture modes.

1-2-1-5. Crack tip plastic zone

The stress at crack tip is limited at least the yield strength of the material, and hence linear elasticity can not be assumed with a certain distance of the crack tip. This non linear region is some times called “ crack tip plastic zone “ [8]. For the ideally elastic mode \( I \) opening stress distribution in the crack plane \( (\theta = 0) \) and in the \( K \) dominant region is:
The elastic analysis becomes inaccurate as the plastic zone ahead of crack tip grows. And the size of this region can be estimated by:

- The Irwin method

The stress in linear elastic material is given by eq (1-24). As the first approximation, we can assume that, the boundary condition between elastic and plastic behavior starts when \( \sigma_y = \sigma_{YS} \), then the first order estimate of plastic zone:

\[
\sigma_y = \frac{K_l}{2\pi r_f} \tag{1-24}
\]

Figure (1-10), the first order and second order estimates of plastic zone \((r_y, r_p)\) [5]

\[
r_y = \frac{1}{2\pi \left( \frac{K_l}{\sigma_{YS}} \right)^2} \quad \text{for plane stress} \tag{1-25}
\]

\[
r_p = \frac{1}{\theta \pi \left( \frac{K_l}{\sigma_{YS}} \right)^2} \quad \text{for plane strain} \tag{1-26}
\]
Second order estimates of the plastic zone size ($r_p$):

$$r_p = \frac{1}{\sigma \pi \left(\frac{K}{\sigma_{YS}}\right)^2}$$

(1-27)

- The strip yield model

The strip model proposed by Dugdal [9] and Barenbatt [10] among others estimates the size of the yield zone ahead of crack tip in a thin plate (plane stress) of elastic perfectly plastic material. Two elastic solutions are superimposed; one through a crack in an infinite plate under remote tension, and the other through a crack with closure stress at crack tip, as in figure(1-11)

![Diagram](a) and (b)

Figure(1-11), the strip yield model[5].

The model assumes that, along the slender plastic zone at the crack tip with length $r_p$, i.e. the total crack length is $2a + 2r_p$. The stress over $r_p$ is $\sigma_{YS}$, and since the stress finite is in the plastic zone, there can not be singularity at the crack tip. This is accomplished by choosing the plastic zone length such that the stress intensity factor from the remote tension and closure stress cancel each other out. This leads to:
1-2-1-6. Crack resistance

The assumption that all strain energy is available for surface energy of new crack faces does not apply to a ductile material where other energy dissipative mechanisms exist. Irwin and Orowan [11] modified Griffith’s equation to take into account the non-reversible energy mechanisms associated with the plastic zone by simply including this term in the original Griffith’s equation:

\[ \tau_p = \frac{R}{H \left( \frac{K_I}{Y} \right)^n} \]  

(1-28)

The right hand of eq (1-29), is given the symblo \( R \) and called the crack resistance. At the point where the Griffith criterion is met. The crack resistance indicates the minimum amount of energy required for crack extension. The energy is called the “work of fracture”, which is the measure of toughness.

Ductile materials are tougher than brittle materials because they can absorb energy in a plastic zone. By contrast, brittle material can only dissipate stored elastic strain energy surface which are created [4].

1-2-1-7. \( K_{IC} \) the critical value of \( K_I \)

The stress intensity factor is \( K_I \) a “scale factor”, which characterizes the magnitude of the stress at some coordinates ( \( \theta = 0 \) ) near the crack tip (theoretically infinite for perfectly elastic materials but limited in practice by plastic deformation), the value of \( K_I \) at the of crack extension is called the critical value: \( K_{IC} \).

\( K_{IC} \) then defines the onset of the crack extension. It does not necessarily indicate fracture of specimen, this depends on the crack stability. \( K_{IC} \) defines the onset of
crack extension, whether this is stable or unstable depends on the crack system. Catastrophic fracture occurs when the equilibrium condition is unstable [4].

1-2-2- elastic-plastic fracture mechanics.

Many of the engineering applications of fracture mechanics have been centred around linear-elastic fracture mechanics (LEFM). This concept become inappropriate when ductile material is used. Much experimentation and analytical effort has been devoted to the development of the elastic plastic fracture mechanics (EPFM) concept. Many EPFM assess the toughness of metallic materials and to predict failure of cracked structural components.

Two alternative parameters characterizing the state at a crack tip are well established in elastic plastic fracture mechanics (EPFM). The first one is the \( J \) integral proposed by J.R.Rice (1968) and Begley and Lands (1972), which represents the intensity of stress or strain rather than the energy release rate. The second one is the crack tip opening displacement (CTOD) \( \delta_T \) as a measure of the state of deformation at crack tip, which dates back to A.H.Cottrell and A.A. Wells (1963) [12].

1-2-2-1. crack tip opening displacement (CTOD).

This parameter of fracture toughness was developed by Wells [13] who discovered that several structural steels could not be characterized by linear elastic fracture mechanics (LEFM), i.e. \( K_I \) was not applicable. He also discovered while examining the fracture surface that the crack surface moves apart prior to fracture. Plastic deformation precedes the fracture and the initially sharp tip is blunted. The plastic deformation increase with increasing fracture toughness, and Wells proposed that the opening at crack tip as a fracture toughness parameter.

The CTOD has no unique definition, figure (1-12) shows two different definitions of the COTD.
Figure (1-12), alternative definition of CTOD, a- displacement at original crack tip 
 b-displacement at the intersection of a $90^\circ$ vertex with the crack flanks [5].

- the Irwin approximation.

When the crack tip is plastically deformed, the crack behaves as if it is longer than the actual crack tip. This is shown by Irwin. [14]. It possible to estimate the crack tip opening displacement in the small scale yielding (SSY), figure (1-13).

The plastic zone correction according to Irwin is:

$$r_y = \frac{1}{2\pi \frac{K}{Y_P}}$$

(1-30)
Eq (1-30), combined with the elastic solution for the displacement of the crack surface in plane stress:

\[ u_y = \frac{k + 1}{\mu} K_I \sqrt{\frac{r_y}{2\pi}} \]  

(1-31)

Gives the CTOD for a stationary crack in small scale yielding

\[ \text{CTOD} = 2u_y \frac{\sigma_{fr}^2}{\pi \sigma_y E} \]  

(1-32)

- The strip yield model.

The strip model provided an alternative analyzing for crack tip opening displacement [15]. Crack tip opening can be defined at the end of strip-yield zone as illustrated in figure (1-14).

![Figure (1-14). Estimation of CTOD from strip yield model](image)

choosing the plastic zone length such that, the stress intensity factor from the remote tension and closure stress cancel each other. This leads to:
The CTOD from the strip model can be derived at the crack tip, by superposition of the crack surface displacement. The CTOD become:

$$CTOD = \frac{B_{55}}{nE} \ln \sec \left( \frac{\pi}{2\nu_{YS}} \right)$$

A series expansion of the logarithmic term in eq (1-34), and truncating all, but first two terms gives (zero will be obtained if only one term is included)
\[ CTOD = \frac{\pi^4 \sigma^4 \alpha^4}{8 \sigma_{YS}^2} = \frac{K_I^4}{\sigma_{YS} E} \]  

The \( CTOD \) from the strip model differs only slightly from eq (1-23), given by the Irwin model.

'1-2-2-2. Crack tip opening angle (CTOA).

The crack tip opening angle (CTOA), it has no clear definition. Figure (1-16), shows the usual definition of the measurement.

![Figure (1-16), definition of CTOA [17]](image)

The measure is traditionally used for crack growth under strictly increased loading, but may also be a decent measure for fatigue crack growth, the measure of \( d \) in figure (1-13), is usually between (0.25-1mm).

1-2-2-3. the J contour integral.

The J contour integral introduced by Rice [18]. As a fracture characterizing parameter for non-linear materials, figure (1-17) shows uniaxial stress-strain behavior of elastic-plastic material and nonlinear material.
Figure (1-17), stress-strain behavior of elastic-plastic and non-linear elastic material[5].

An analysis assumes that non-linear elastic behavior may be valid for elastic-plastic material. Rice applied deformation plasticity to the analysis of crack in a non-linear material. He showed that the non-linear energy release rate $J$ could be written as a path-independent line integral. Hutchinson [19] and Rice and Rosengren[20] also showed that $J$ uniquely characterize crack tip stresses and strain in non-linear material.

- $J$ as a path-independent line

Rice (1968), developed a powerful mathematical device, Namely Rice line integral $J$, to describe the energy flow into the crack tip per unit fractured area, its given by:

$$J = \int_T \left[ wdy - T_{li} \frac{\partial u_l}{\partial x_i} \right] ds$$

(1-36)
Figure (1-18), arbitrary contour around the tip of crack[5].

Where,

\(w\) = strain energy density.

\(T_t\) = components of the traction vector.

\(u_t\) = displacement vector components

\(ds\) = length increment along the contour \(\Gamma\).

the strain energy density is defined as:

\[ w = \int \sigma_{ij} \epsilon_{ij} ds \]

Where, \(\sigma_{ij}\) and \(\epsilon_{ij}\) are the stress and strain respectively. The traction is a stress vector at a given point on the contour. That is, if we were to construct a free body diagram of material inside inside of the contour, \(T_t\) would define the stress acting at the boundaries.

The components of traction vector are given by:

\[ T_t = \sigma_{ij} n_j \]

Where, \(n_j\) are the components of unit vector normal to \(\Gamma\). Rice showed that the value of the J integral is independent of the path of integration around the crack tip.

- \(\Delta J\) integral path
Lamba[21] in 1975, presented $\Delta J$ integral, when the material in front of crack tip experienced a cyclic stress-strain, and consequently the process is characterized by $\Delta \sigma_{ij}$ and $\Delta \epsilon_{ij}$ [16]. Figure (1-19) shows a cyclic stress-strain loop, where the initial value is indicated by number 1 and the final value by number 2, where the $J$ integral was then defined as [22,23,24].

![Figure (1-19), stress-strain curve][5]

$\Delta J = \int_{\Gamma} \left[ (\psi(\Delta \epsilon_{ij})) dy - \Delta T_{ij} \left( \partial u_i \partial y / \partial x \right) ds \right]$

(1-37)

Where, $\Gamma$ is the integration bath around the crack tip, $\Delta T_{ij}$ and $\Delta u_i$ are the changes In traction and displacement between point 1 and point 2, $\psi$ corresponds to strain energy density and defined as:

$\psi(\Delta \epsilon_{ij}) = \int_{\epsilon_{ij}}^{\epsilon_{ij}^{(2)}} \alpha_{ij} d\epsilon_{ij} = \int_{\epsilon_{ij}}^{\epsilon_{ij}^{(2)}} \left[ (\sigma_{ij} - \sigma_{ij}^{2}) \right] d\epsilon_{ij}$

(1-38)
The requirements for $\Delta J$ to be path independent as are analogous to the original J-integral mentioned above, i.e. $\sigma_{ij} = \frac{\partial W}{\partial \varepsilon_{ij}}$

$\Delta J$ calculation is given by:

$$\Delta J = \frac{\eta}{Bb} \int_{0}^{\Delta V} \Delta P \, dV = \frac{\eta}{Bb} \int_{V_{\text{min}}}^{V_{\text{max}}} (P - P_{\text{min}}) \, dv$$

(1-39)

Where, $\eta$ dimensionless constant, B the specimen thickness, and b the uncracked ligament, $P_{\text{max}}, P_{\text{min}}, V_{\text{max}}, V_{\text{min}}$ are the maximum load and displacement respectively, during the specific load cycle as shown in figure (1-20).

![Figure 1-20](image)

Figure (1-20), integration area (a)-without respect to the crack closure level (b)- with respect to the crack closure level[5].

1-2-2-3. HRR Singularity

The relation between J and CTOD is also investigated by Shih [25]. He evaluates the displacement at the crack tip given by HRR singularity and related those to J and flow properties, the displacement near crack tip is given by eq (1-31), according to the HRR solution[16].

$$u_{\varepsilon} = \frac{\alpha \sigma_{0}}{E} \left( \frac{E}{\alpha \sigma_{0}^{1-\nu}} \right)^{\frac{1}{1-\nu}} r^{\frac{1}{1-\nu}} (\theta, n)$$

(1-40)
Here $\delta_i$ is a dimensionless function of $(\sigma, n)$, where $n$ is the strain hardening exponent. Shih evaluates the CTOD at the 90º intersection as defined by Rice [18].

$$CTOD = d_n \frac{J}{\sigma_o}$$  \hspace{1cm} (1-41)

Where, $d_n$ constant dimensionless given as shown in fig (1-21).

Figure (1-21), the dimensionless constant $d_n$ for plane stress and plane strain [5].

Shih showed that, the relationship of eq (1-31), applies well beyond the validity of LEFM for stationary racks.

Rice [26]. Presented the CTOD for quasi-statically growing crack in a perfectly plastic material loaded in mode $I$.
Here $R_e$ is the plastic zone size. The strain exhibit a logarithmic singularity while the stresses are bounded. Resulting in no energy flow to the crack tip[16].
2-Finite element analysis of solids

2-1. Introduction

The finite element method (FEM) is a numerical technique for solving problems which are described by partial differential equations or can be formulated as a functional minimization. A domain of interest is represented as an assembly of finite elements. Approximating functions in finite elements are determined in terms of nodal values of a physical field which is sought. A continuous physical problem is transferred into discretized finite element problem with unknown nodal values.

2-2. Formulation of finite element equation

Several approaches can be used to transform the physical formulation of the problem to its finite element discrete analogue. If the physical formulation of the problem is a differential equation then the most popular method of its finite element formulation is the Galerkin method. If the physical problem can be formulated as a minimization of a functional then variational formulation of the finite element is usually used [27].

2-3. Linear-elastic finite element analysis

Let us consider a three-dimensional elastic body subjected to surface and body force. In addition, displacement is specified on some surface area. For a given geometry of the body, applied force, displacement boundary condition, and material stress-strain law it is necessary to determine the displacement field for the body [27].

- Displacement vector $\{u\}$, along coordinates axes $X$, $Y$ and $Z$

$$\{u\} = \{u \ v \ w\} \quad (2-1)$$

- Strain vector $\{\varepsilon\}$ :

$$\{\varepsilon\} = \{\varepsilon_x \ \varepsilon_y \ \varepsilon_z \ \varepsilon_{xy} \ \varepsilon_{xz} \ \varepsilon_{yz}\} \quad (2-2)$$

- Strain-displacement relationship:

$$\{\varepsilon\} = [D] \{\delta\} \quad (2-3)$$

Where $[D]$ is the matrix differentiation Operator:
\[ \mathbf{W} = \begin{bmatrix} \frac{\partial}{\partial x} & u & 0 \\ \frac{\partial}{\partial y} & 0 & u \\ \frac{\partial}{\partial z} & 0 & 0 \end{bmatrix} \] (2-4)

-Stress vector \( \mathbf{\sigma} \):

\[ \mathbf{\sigma} = \begin{bmatrix} \sigma_x & \sigma_y & \sigma_z & \sigma_{xy} & \sigma_{xz} & \sigma_{yz} \end{bmatrix} \] (2-5)

-Stress-strain relationship of elastic body (Hook’s law):

\[ \mathbf{\sigma} = [\mathbf{E}] \mathbf{\epsilon} \] (2-6)

Where \( [\mathbf{E}] \) is the elasticity matrix:

\[ [\mathbf{E}] = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \] (2-7)

\[ \lambda = \frac{v\mu}{1 + v(1 - 2v)} \]
\[ \mu = \frac{E}{2(1 + 2\nu)} \]

Where:

- \( E \) = Young’s modulus
- \( \nu \) = Poisson’s ratio

The purpose of FEA solution of elastic problem is to find such displacement field which provides minimum to functional of total potential energy:

\[ \prod_{E} \left[ -\frac{1}{2} \int_{V} \mathbf{u}^T \mathbf{F} \mathbf{u} \, dv - \int_{V} \mathbf{u}^T \mathbf{p}^V \, dv - \int_{S} \mathbf{u}^T \mathbf{p}^S \, dS \right] \]  

(2-8)

Where:

- \( \mathbf{p}^V = \{ \mathbf{p}_x^V \mathbf{p}_y^V \mathbf{p}_z^V \} \) Vector of body force \hspace{1cm} (2-9)
- \( \mathbf{p}^S = \{ \mathbf{p}_x^S \mathbf{p}_y^S \mathbf{p}_z^S \} \) Vector of surface force \hspace{1cm} (2-10)

2-3-1. Three dimensional isoparametric elements

- Shape function:

Hexahedral (or brick type) linear-8 nodes and quadratic 20-nodes three-dimensional elements are illustrated in fig 2-1, the term isoparametric means that geometry and displacement field are specified in parametric form and are interpolated with the same
Figure (2-1), linear - quadratic elements and their representation in local coordinate [28]

Function Shape [27], function used for interpolation are polynomials of local coordinates \( \xi, \eta, \) and \( \zeta \) \((-1 \leq \xi, \eta, \zeta \leq 1\). Both coordinates and displacement are interpolated with the same function: \( \{\mathbf{q}\} = [\mathbf{N}] \{\mathbf{u}\}\)

\[
\{\mathbf{q}\} = \{u \ v \ w\} \\
\{\mathbf{q}\} = \{u_1 v_1 w_1 u_2 v_2 w_2 \ldots\} \\
\{\mathbf{x}\} = [\mathbf{N}] \{\mathbf{x}\} \\
\{\mathbf{x}\} = \{x \ y \ z\} \\
\{\mathbf{x}\} = \{x_1 y_1 z_1 x_2 y_2 z_2 \ldots\}
\]

Here \( u, v, w \) are displacements at point at point with local coordinate \( \xi, \eta, \zeta \); \( u_1, v_1, w_1, \ldots \) are displacement values at nodes \( x_1, y_1, z_1 \); \( x_2, y_1, z_1, \ldots \) are point coordinates and \( x_0, y_0, z_0, \ldots \) are coordinates of nodes. The matrix of shape function is:

\[
[N] = \begin{bmatrix}
N_1 & 0 & 0 & N_2 & 0 & 0 & \ldots \\
0 & N_1 & 0 & 0 & N_2 & 0 & \ldots \\
0 & 0 & N_1 & 0 & 0 & N_2 & \ldots \\
\end{bmatrix}
\]
Shape function of linear element is equal to:

\[ N_l = \frac{3}{8}(1 + \xi \zeta)(1 + \eta_0)(1 + \eta_0) \]
\[ \xi_0 = x x, \eta_0 = n n, \zeta_0 = z z \]  
(2-14)

Shape function of quadratic element with 20 nodes can be written as:

\[ N_l = \frac{1}{8}(1 + \xi \zeta)(1 + \eta_0)(1 + \eta_0) \]
\[ l = 2, 6, 14, 18 \]  
(2-15)

\[ N_l = \frac{1}{4}(1 - \xi \zeta)(1 + \eta_0)(1 + \eta_0) \]
\[ l = 4, 8, 16, 20 \]

\[ N_l = \frac{1}{4}(1 - \xi \zeta)(1 + \eta_0)(1 + \eta_0) \]
\[ l = 9, 10, 11, 12 \]

In the above relation \( \xi, \eta_0, \zeta_0 \) are values of local coordinates \( \xi, \eta, \zeta \) at nodes.

-Strain displacement matrix:

The strain vector \( \mathbf{\varepsilon} \) contains six different components of strain tensor

\[ \mathbf{\varepsilon} = \{\varepsilon_x, \varepsilon_y, \varepsilon_z, \varepsilon_{xy}, \varepsilon_{xz}, \varepsilon_{yz}\} \]  
(2-16)

The strain-displacement matrix has the following appearance:

\[ [B] = [D][N][B_1B_2B_3... \]
Derivatives of shape function with respect to global coordinates are obtained as follows:

\[
B_i = \begin{bmatrix}
\frac{\partial N_i}{\partial x} & 0 & 0 \\
0 & \frac{\partial N_i}{\partial y} & 0 \\
0 & 0 & \frac{\partial N_i}{\partial z}
\end{bmatrix}
\]

(2-17)

\[
\begin{bmatrix}
\frac{\partial N_i}{\partial x} \\
\frac{\partial N_i}{\partial y} \\
\frac{\partial N_i}{\partial z}
\end{bmatrix} - [J]^{-1} \begin{bmatrix}
\frac{\partial N_i}{\partial \xi} \\
\frac{\partial N_i}{\partial \eta} \\
\frac{\partial N_i}{\partial \zeta}
\end{bmatrix}
\]

(2-18)

Where the Jacobian matrix has the following appearance:

\[
[J] = \begin{bmatrix}
\frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \zeta} \\
\frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} & \frac{\partial y}{\partial \zeta} \\
\frac{\partial z}{\partial \xi} & \frac{\partial z}{\partial \eta} & \frac{\partial z}{\partial \zeta}
\end{bmatrix}
\]

(2-19)
The partial derivative of \( x, y, z \) with respect to \( \xi, \eta, \zeta \) are found by differentiation of displacements expressed through shape functions and nodal displacement values:

\[
\frac{\partial x}{\partial \xi} = \sum \frac{\partial N_l}{\partial \xi} x_l, \quad \frac{\partial x}{\partial \eta} = \sum \frac{\partial N_l}{\partial \eta} x_l, \quad \frac{\partial x}{\partial \zeta} = \sum \frac{\partial N_l}{\partial \zeta} x_l
\]

\[
\frac{\partial y}{\partial \xi} = \sum \frac{\partial N_l}{\partial \xi} y_l, \quad \frac{\partial y}{\partial \eta} = \sum \frac{\partial N_l}{\partial \eta} y_l, \quad \frac{\partial y}{\partial \zeta} = \sum \frac{\partial N_l}{\partial \zeta} y_l
\]

\[
\frac{\partial z}{\partial \xi} = \sum \frac{\partial N_l}{\partial \xi} z_l, \quad \frac{\partial z}{\partial \eta} = \sum \frac{\partial N_l}{\partial \eta} z_l, \quad \frac{\partial z}{\partial \zeta} = \sum \frac{\partial N_l}{\partial \zeta} z_l
\]

(2-20)

The transformation of integrals from the global coordinates system to local coordinate system is performed with the use of determinant of Jacobian matrix:

\[
dV = dx dy dz = |J| d\xi d\eta d\zeta \quad (2-21)
\]

- Element properties

Element equilibrium equation has the following form:

\[
[K][\psi] = [\psi] \quad (2-22)
\]

Element matrices and vectors:

Stiffness matrix

\[
[K] = \int [B]^T [E] [B] dV \quad (2-23)
\]

Force vector (volume and surface loads):
\[ [P] = \int_N [N]^T [P^V] dV + \int_S [N]^T [P^S] dS \]  \hspace{1cm} (2-24)

The elasticity matrix \([E]\) is:

\[ [E] = \begin{bmatrix}
\lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\
\lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\
\lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\
0 & 0 & 0 & \mu & 0 & 0 \\
0 & 0 & 0 & 0 & \mu & 0 \\
0 & 0 & 0 & 0 & 0 & \mu
\end{bmatrix} \]  \hspace{1cm} (2-25)

Where \(\lambda\) and \(\mu\) are elastic constants, which can be expressed through the elasticity modulus \(E\) and Poisson’s ratio:

\[ \lambda = \frac{vE}{(1+v)(1-2v)}, \quad \mu = \frac{E}{2(1+v)} \]  \hspace{1cm} (2-26)

Integration of the stiffness matrix

Integration of the stiffness matrix for three-dimensional isoparametric elements is carried out in the local coordinate \(\xi, \eta, \zeta\):

\[ [K] = \int_{\xi_1}^{\xi_2} \int_{\eta_1}^{\eta_2} \int_{\zeta_1}^{\zeta_2} [B(\xi, \eta, \zeta)]^T [E(\xi, \eta, \zeta)] [B(\xi, \eta, \zeta)] d\xi d\eta d\zeta \]  \hspace{1cm} (2-27)

Usually \(2 \times 2 \times 2\) integration is used for linear elements integration and integration \(3 \times 3 \times 3\) applied to the evaluation of the stiffness matrix for quadratic elements.
- Calculation of strains and stress:

After computing elements matrices and vectors, the assembly process is used to compose the global equation system. Solution of the global equation system provides displacements at nodes of the finite element nodal. Using disassembly nodal displacement for each element can be obtained.

Strains inside an element are determined with the use of displacement, differentiation matrix:

\[
\{\varepsilon\} = [B]\{u\} \tag{2-28}
\]

Stresses calculated with the hook's law:

\[
\{\sigma\} = [E]\{\varepsilon\} \tag{2-29}
\]

The highest precision for displacement gradients are at the geometric center for linear element, and at reduced integration points \(2 \times 2 \times 2\) for quadratic hexagonal element.

2-4. Elastic –plastic finite element analysis:

The elastic-plastic stress analysis of solids which conform to plane stress or plane strain conditions is considered. Only the essential expressions will be reproduced here for theoretical and numerical treatment. The basic laws governing elastic-plastic continuum behavior are summarized before considering numerical formulation. In particular, the form of the yield criterion which governs the onset of plastic flow must be defined as well as the incremental relationship between stress and strain during continuing elastic-plastic deformation. In this section the Von Misses yield criteria, which closely approximate metal plasticity behavior are considered. The basic theoretical expression is then rewritten in a form suitable for numerical manipulation.
2-4-1. The mathematical theory of plasticity:

In order to formulate a theory which models elastic-plastic material deformation three requirements have to be met:

An explicit relationship between stress and strain must be formulated to describe material behavior under elastic conditions, i.e. before the onset of plastic deformation.

A yield criterion indicating the stress level at which plastic flow commences must be postulated.

A relationship between stress and strain must be developed for post-yield behavior, i.e. when the deformation is made up of both elastic and plastic components.

2-4-2. The yield criteria:

The yield criterion determines the stress level at which plastic deformation begins and can be written in the general form

\[ f(\sigma) = f(J_2, J_3) = k(k) \]  \hspace{1cm} (2-30)

Where \( f \) is some function of the deviatoric stress invariants:

\[ J_2 = \frac{1}{2} \sigma_{ij} \sigma_{ij} \]
\[ J_3 = \frac{1}{3} \sigma_{ij} \sigma_{jk} \sigma_{ki} \]  \hspace{1cm} (2-31)

In which \( \sigma_{ij} \) the deviatoric stress components
\[
\sigma'_{ij} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk}
\]  \hspace{1cm} (2-32)

The term \(k\) in (2-30) can be a function of a hardening parameter. The two most common yield criteria employed in the description of the behavior of metals are the Tresca criterion and the Von Misses criterion.

2-4-3. The Von Misses criterion:

Von Misses suggested that yielding occurs when \(J_2'\) reaches a critical value, or

\[
(J_2')^{\frac{1}{2}} = k(\kappa)
\]  \hspace{1cm} (2-33)

In which \(k\) is a material parameter to be determined the second deviatoric stress invariant, \(J_2'\) can be explicitly written as

\[
J_2' = \frac{1}{2} \sigma_{ij} \sigma_{ij} = \frac{1}{6} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]
\]

\[
= \frac{1}{2} \left[ \sigma_x^2 + \sigma_y^2 + \sigma_z^2 \right] + \tau_{xy}^2 + \tau_{yx}^2 + \tau_{yz}^2 + \tau_{zy}^2
\]  \hspace{1cm} (2-34)

Yield criterion (2-33) may be further written as

\[
\bar{\sigma} = \sqrt{3}(J_2')^{\frac{1}{2}} = \sqrt{3}k
\]  \hspace{1cm} (2-35)

Where:
\[ \bar{\sigma} = \sqrt{\frac{1}{2} \left\{ \sigma_\theta^2 \sigma_\phi^2 \right\}} \]

(2-36)

Figure (2-2), Von Misses and Tresca yield surface in principle stress coordinate [29]

And \( \bar{\sigma} \) is termed the effective stress, generalized stress or equivalent stress, Figure (2-2) shows the geometrical interpretation of the Von Misses yield surface to be a circular cylinder whose projection onto the \( \pi \) plane is a circle of radius \( \sqrt{2k} \). A physical meaning of the constant \( k \) can be obtained by considering the yielding of materials under simple stress states. The case of pure shear (\( \sigma_1 = -\sigma_2, \sigma_3 = 0 \)) requires use of (2-33) and (2-35) that \( k \) must equal the yield shear stress. Alternatively the case of unaxial tension (\( \sigma_2 = \sigma_3 = 0 \)) requires that \( \sqrt{3}k \) is the unaxial yield stress.
The Tresca yield locus is a hexagon with distances of $\sqrt{\frac{2}{3}}Y$ from origin to apex on the $\pi$ plane whereas the Von Misses yield surface is a circle of radius $\sqrt{2}k$. By suitably choosing the constant $Y$, the criteria can be made to agree with each other, and with experiment, for a single state of stress. This may be selected arbitrarily: it is conventional to make the circle pass through the apices of the hexagon by taking the constant $Y = \sqrt{3}k$, the yield stress in simple tension. The criteria then differ most for a state of pure shear, where the Von Misses criterion gives a yield stress $2 / \sqrt{3}(= 1 \cdot 15)$ times that given by the Tresca criterion.

2-4-4. Work or strain hardening:

After initial yielding, the stress level at which further plastic deformation occurs may be dependent on the current degree of plastic straining. Thus the yield surface will vary at each stage of the plastic deformation with the subsequent yield surfaces being dependent on the plastic strains in some way. In this text attention is restricted to an isotropic hardening model, in which the original yield surface expands uniformly without translation. The progressive development of the yield surface can be defined by relating the yield stress $k$ to the plastic deformation by means of the hardening parameter $\kappa$. In a work hardening hypothesis $\kappa$ related to the total plastic work $W_p$ as

$$\kappa = W_p = \int \sigma \left( \epsilon_p \right)_p$$

(2-37)

In which $\left( \epsilon_p \right)_p$ are the plastic components of strain occurring during a strain increment.

Alternatively, in a strain hardening hypothesis, $\kappa$ is related to a measure of the total plastic deformation termed the effective or equivalent plastic strain which is defined incrementally as

$$\kappa = \bar{\epsilon}_p$$
Then

\[ d\varepsilon_p = \sqrt{\frac{2}{3}} \left\{ (d\varepsilon_{ij})_p (d\varepsilon_{ij})_p \right\}^{1/2} \]  \hspace{1cm} (2-38)

Where \( \varepsilon_p \) is the result of integrating \( d\varepsilon_p \) over the strain path.

2-4-5. Elastic-plastic stress/strain relation:

After initial yielding the material behavior will be partly elastic and partly plastic. During any increment of stress, the changes of strain are assumed to be divisible into elastic and plastic components, so that

\[ d\varepsilon_{ij} = (d\varepsilon_{ij})_e + (d\varepsilon_{ij})_p \]  \hspace{1cm} (2-39)

The elastic strain increment is related to the stress increment by the incremental form of (2-40).

In order to derive the relationship between the plastic strain component and the stress increment a further assumption on the material behavior must be made. In particular it will be assumed that the plastic strain increment is proportional to the stress gradient of a quantity termed the plastic potential \( Q \), so that

\[ (d\varepsilon_{ij})_p = d\lambda \frac{\partial Q}{\partial \sigma_{ij}} \]  \hspace{1cm} (2-40)

Where \( d\lambda \) is a constant termed the plastic multiplier. Equation (2-40) is termed the flow rule since it governs the plastic flow after yielding. The potential \( Q \) must be a function of \( J'_2 \) and \( J'_3 \) but as yet it cannot be determined in its most general form. The assumption \( \equiv Q \) gives rise to an associated theory of plasticity. In this case (2-40) becomes
\[
(d\varepsilon_{ij})_p = d\lambda \frac{\partial f}{\partial \sigma_{ij}}
\]  

(2-41)

And is termed the normality condition since \( \partial f / \partial \sigma_{ij} \) is a vector directed normal to the tiled surface at the stress point under consideration.

**2-3-6. Matrix formulation:**

The theoretical expression reviewed in Section (2-31) will now be converted to matrix form. The yield function, defined in (2-31), can be rewritten as

\[
F(\sigma, \kappa) = f(\sigma) - k(\kappa) = 0
\]  

(2-42)

In which \( \sigma \) is the stress vector and \( \kappa \) is the hardening parameter which governs the expansion of the yield surface. The differential form of (2-42) is

\[
a^T d\sigma - A d\lambda = 0
\]  

(2-43)

In which

\[
a^T = \frac{\partial F}{\partial \sigma} = \left\{ \frac{\partial F}{\partial \sigma_x}, \frac{\partial F}{\partial \sigma_y}, \frac{\partial F}{\partial \tau_{xy}}, \frac{\partial F}{\partial \sigma_z} \right\}
\]  

(2-44)

And
\[ A = -\frac{1}{\partial \lambda} \frac{\partial F}{\partial k} \]  

(2-45)

The vector \( \mathbf{a} \) is termed the flow vector. Substituting from (2-43) into (2-41) result in

\[ d \mathbf{e} = [ \mathbf{D} ]^{-1} d \mathbf{\sigma} + d \lambda \frac{\partial F}{\partial \mathbf{\sigma}} \]  

(2-46)

Manipulation of (2-43) and (2-46) leads to the following complete elastic-plastic incremental stress-strain relation

\[ d \mathbf{\sigma} = \mathbf{D}_{\text{ep}} d \mathbf{e} \]  

(2-47)

With

\[ \mathbf{D}_{\text{ep}} = \mathbf{D} - \frac{d_{\delta}^T d_{\delta}}{A + d_{\delta}^T d_{\delta}} ; \quad d_{\delta} = \mathbf{D} \mathbf{a} \]  

(2-48)

Assumption of a work hardening hypothesis and consideration of unaxial loading conditions result in the scalar term \( A \) being given by

\[ A = H = \frac{d \mathbf{\sigma}}{d \mathbf{\epsilon}_p} = \frac{E_T}{1 - E_T / E} \]  

(2-49)

In which \( E_T \) is the elastic-plastic tangent modulus of the unaxial stress-strain curve, and \( E \) is the elastic modulus of the material.

2-5 ABAQUS software package
ABAQUS software is developed by Hibbitt, Karlsson and Sorensen, Inc [30]. It is a complete package of powerful engineering simulation programs, based on the finite element analysis. This simulation software is capable of performing a simple linear analysis to the most complex non-linear simulation. ABAQUS -standard and ABAQUS Explicit are two main modules available in ABAQUS.

ABAQUS - Standard:

ABAQUS - Standard is an all purpose analysis module that can solve a variety of problems covering linear and non-linear problems maintaining the accuracy and reliability of the results. And it consists of three distinct stages, preprocessing, simulation and post processing.

ABAQUS - Explicit:

ABAQUS - Explicit is a special purpose to analysis module that uses dynamic finite element formulation which is applied to deal with transient and dynamic in nature.

ABAQUS - CAE:

ABAQUS - CAE is the total ABAQUS working interface that includes all the options to generate ABAQUS module, to submit and monitor the job for analysis and also a means to review the results.

2-5-1. Elastic-plastic analysis in ABAQUS:

Stress-strain follows Hook’s law, giving a linear relationship at low strain values which is true for most materials, but at higher strain the material yield. At which point the material relationship becomes non-linear and irreversible, and can be described as a material nonlinearity. Newton-Raphson method is used in ABAQUS to obtain solution for a non-linear problem by applying the specified loads gradually and incrementally the solution is found reaching towards the final solution. ABAQUS breaks the analysis into a number of load increments and finds the approximate equilibrium configuration at the end of each load increment. Hence it often takes ABAQUS several iterations for a defined
loading condition. The sum of all of the incremental responses is the approximate solution for the nonlinear analysis [31].

Figure (2-4), traditional Newton-Raphson method vs. arc-length method [32]
Structural integrity assessment of the pressure vessel

3-1 Introduction

Fracture mechanics-based structural integrity assessment or fitness-for service (FFS) is not a new concept. The nuclear and offshore oil and gas industries were the main drivers behind the development of the FFS procedure. Fracture mechanics methods have been used to assess the structural integrity of pressure equipment for many years. Fracture describes a failure mechanism that involves the propagation of a crack. How that crack propagates depends on three variables: flaw size, material properties and stress state at the region of the flaw.

Irwin’s stress intensity approach fracture occurs when the stress intensity factor at the crack tip exceeds the material fracture toughness. Linear elastic fracture mechanics introduce the concept of a stress intensity factor $K$. The stress intensity factor is a single parameter that represents the crack driving force and characterizes the stress field at the crack tip. LEFM is only valid for brittle materials. For ductile materials, the local stress-state close to the crack tip is such that plasticity occurs, and when this is significant, then $K$ is no longer appropriate and means to account for plasticity at crack tip. Additional parameters are required to characterize fracture toughness, such as the crack tip opening displacement (CTOD, $\delta$) which is a strain based parameter, and the J-integral which is energy based parameter.

Fitness-for-service is performed after a defect or crack has been found following routine inspection, maintenance or safety checks, or when the effect of an undetected crack needs to be considered. The assessment determines whether the pressure vessel is safe to operate with the defect or to establish inspection intervals for monitoring the defect. If the defect size is unacceptable, then the user must be decided whether to repair it, or replace the equipment. It is possible to represent material properties, defect geometry and loading conditions in a mathematical form, and generate what is known as a failure assessment diagram (FAD). The FAD approach can be considered as independent from component geometry.
3-2. Failure Assessment Diagram approach (FAD).

A failure assessment diagram represents a two-parameter approach. For fracture to occur the stress intensity factor at crack tip must be greater than the material toughness or critical stress intensity factor ($K_{IC}$). However, plastic collapse can also occur if the stress is high relative to the ultimate tensile strength (UTS) of material. A typical failure assessment diagram is shown in figure (2-1).

![Failure Assessment Diagram](image)

Figure (3-1), failure assessment diagram FAD [33].

A vertical axis of the failure assessment diagram represents the criteria for brittle or ductile fracture, often known as the fracture toughness ratio ($K/I_c$) which is the ratio of stress intensity factor ($K$) to material fracture toughness ($I_c$). The horizontal axis represents the likelihood of plastic collapse, often known as the load ratio ($L_p$).

3-3. R-Curve approach
In order to construct R-curve for a material, where a toughness parameter such as K, J, or (CTOD) is plotted against the crack extension, a fracture toughness test is performed to measure the resistance of material to a crack extension. A variety of organizations publish standardized procedures for fracture toughness measurement, including the American Society for Testing and Materials (ASTM), the British Standards Institution (BSI), the International of Standards (ISO) and Japan Society of Mechanical Engineers (JSME).

3-3-1.Specimen configuration

There are five types of specimens that are permitted in ASTM standards which characterize fracture initiation and crack growth, the configurations that are currently standardized include the compact specimen, the single-edge notch bend (SEN (B)) geometry, the arc shaped specimen, the disk specimen and the middle tension (MT) panel. Figure (3-2) shows a drawing for each specimen type.

Figure (3-2-a, 3-2-b), standardized fracture mechanics test specimen a- compact specimen, b- disk shaped compact specimen [5].
Figure (3-2-c, 3-2-d, 3-2-e), standardized fracture mechanics test specimen c- single-edge-notch bend SEN (B) specimen, d- middle tension (MT) specimen, e- arc shaped specimen [5].

3-3-2. $K_{IC}$ testing

When a material behaves in a linear elastic manner prior to fracture, such that the plastic zone is small compared to the specimen dimension, a critical value of the mode I stress intensity factor $K_{IC}$ may be an appropriate fracture parameter. ASTM-399 [34] was the first standard test method for $K_{IC}$ testing, other $K_{IC}$ testing was British Standard 5447 [35] are generally based on ASTM-399 [5].
Figure (3-3), three types of load-displacement behavior in $K_{IC}$ test [5].

Displacement and load are monitored during the test of pre-cracked specimen until the fracture of specimen. The critical load $P_Q$ was defined in several ways depending on the type of curve:

- Curve I, load-displacement behavior is smooth and deviates slightly from linearity. This non-linearity could be caused by plasticity, or subcritical crack growth, or both.
- Curve II, a small amount of unstable crack growth occurs before the curve deviates from non-linearity.
- Curve III, behavior fails completely before achieving 5% of non-linearity.

The crack length must be measured from the fracture surface [5].

According to ASTM-399:

$$K_Q = \frac{P_Q}{B \sqrt{W}} f (\alpha_M)$$  \hspace{1cm} (3-1)
Where:

\[ f^{(a/w)} \] = dimensionless function of \( a/w \)

\( W \) = width of the specimen

\( B \) = thickness of specimen

\( a \) = crack length

The \( K_Q \) computed from equation (3-1) is a valid \( K_{IC} \) result if

\[ 0.45 \leq \frac{a}{W} \leq 0.55 \]  

(3-2a)

\[ B, a \geq 2.5 \left( \frac{K_Q}{\sigma_{YS}} \right)^2 \]  

(3-2b)

\[ R_{\max} \leq 1.1 F_Q \]  

(3-2c)
Figure (3-4), the effect of specimen thickness on the fracture toughness in titanium [36].

Figure (3-4), shows the effect of specimen thickness on $K_Q$ in titanium alloy [5]. In this case the ligament length was fixed while thickness was varied.

Figure (3-5), the effect of ligament length on fracture toughness in aluminum alloy [37, 38]

Figure (3-5), shows the effect of ligament length on the fracture toughness of high strength aluminum alloy. Note that the measured fracture toughness increased with ligament length of specimen.

3-3-3. K-R Curve testing
In order to construct K-R curve for material that exhibit ductile crack extension the ASTM standard E 561[39] is recommended.

Figure (3-6), K-R curve; $K_c$ occurs at the point of tangency between the crack driving force and R curve [5].

Figure (3-6), illustrates a typical K-R curve in a predominantly linear elastic material. The R curve is initially very steep, since only little or no crack growth occurs, with increasing $K_1$ (Crack driving force). As the crack begins to grow, $K_I$ increase with the crack growth until steady state is reached, where the R-curve becomes flat. It is possible to define critical stress intensity $K_C$ where the crack driving force tangent to the R-curve [5].

3-3-3-1. Experimental measurements of K-R curves

For computing $K_I$ and crack extension the ASTM standard 561 outlines a number of alternative methods, the appropriate approach depends on the relative size of plastic zone at crack tip. If plasticity is negligible, the behavior of load-displacement curve as shown in figure (3-7) indicates deviation from the initial linear shape because the
compliance is continuously changing and if the specimen unloaded the load-displacement curve it will return to the origin point. The following equation relates instantaneous stress intensity with current crack length and load.

\[ K_I = \frac{P}{B\sqrt{W}} f(\alpha/W) \] (2-3)

Figure (3-7), load-displacement curve of crack growth in the absence of plasticity [5].
Figure (3-8), load-displacement behavior of crack growth with plasticity [5].

In the case of presence of a plastic zone ahead of a tip of a growing crack, the non-linear behavior of the load-displacement curve is caused by plastic deformation and crack growth. If the specimen unloaded before fracture the load-displacement curve will not return to the origin point because crack tip plasticity produces a certain amount of permanent strain in the specimen.

-Determination of physical crack length

The stress intensity factor should be corrected for plastic deformation affect by determining physical crack length, ASTM suggests two an alternative methods, Irwin plastic zone correction approach and secant method [38].

3-3-4. J_e testing

In order to construct J-R curve, ASTM standard E 1820 [40], and the British standard BS 7448: part 1[41] are suggested and they cover this test.

There are two alternative methods of J testing provided by ASTM standard E 1820

- Basic procedure: this method performs by monotonically loading the specimen until fracture or to a certain displacement.
- Resistance curve procedure: in this procedure the growth of crack monitored during the test.

To construct R-curve it is convenient to divide the J into elastic component and plastic as follows according to ASTM E1820:

\[
J = J_{el} + J_{pl}
\]  \hspace{1cm} (3-4)

\[
J_{el} = K^2 \left( \frac{1 - \nu^2}{E} \right)
\]  \hspace{1cm} (3-5)

\[
K = \frac{P}{B \cdot W} f(\alpha, \beta, \gamma)
\]  \hspace{1cm} (3-6)

If side groove specimen are used, then

\[
K = \frac{P}{B \cdot B_{NW} W} f(\alpha, \beta, \gamma)
\]  \hspace{1cm} (3-7)
Figure (3-9), side groove in a fracture mechanics test specimen [5].

ASTM E 1820 including a simplified method for computing J plastic from area under load-displacement curve [42].

\[
J_{pl} = \frac{\eta A_{pl}}{B_N b_o}
\]  

(3-8)

Where:

- \( J_{el} \) = elastic component of J
- \( J_{pl} \) = plastic component of J
- \( K \) = stress intensity factor
- \( \nu \) = poisson’s ratio
- \( E \) = Young’s modulus
- \( b_o \) = the initial ligament length
- \( \eta \) = dimensionless constant
3-3-4-1. J-R Curve

The most common single-specimen test technique is the unloading compliance method, which is illustrated in figure (3-11). The crack length is computed at regular intervals during the test by partially unloading the specimen and measuring the compliance. As the crack grows, the specimen became more compliant (less stiff) [5].
Figure (3-12), J-R curve for A 710 Steel [43]
4. Methodology and approach of FEA and experimental work

4-1. Introduction

The hydrostatic test of a full-scale model of penstock has been modeled in an FEA ABAQUS, to simulate the behavior of the finite element model with inner pressure. In the first portion of analysis the von Misses stress distribution will be investigated in two steps; the first load-unload and the second load-unload, and focus on where the yielding initiates and spreads. For the second portion of the numerical study the behavior of the model with initial residual stresses in weld joints will be analyzed for von Misses stresses distribution and initiation of plasticity for first load-unload and second load-unload.

4-2. Methodology and approach of FEA

4-2-1. Created the geometry of model in ABAQUS/CEA.

The geometry model of penstock has been modeled in ABAQUS/CEA, as illustrated in figure (4-1). This model has been sketched as the experimental model of penstock, as in figure (4-6), except that the third segment of cylindrical mental of the experimental model has been left out, because the effect of size of the geometrical model on the run time of finite element analysis. On the other hand, this part of the experimental model is not important and could be negligible.

Figure (4-1), finite element model of penstock as sketched in ABAQUS/CEA.
4-2-2. Mechanical properties of FE model

To perform elastic-plastic analysis in ABAQUS, elastic and plastic properties are needed. For elastic properties, we need to define Young’s module and Poisson’s ratio in ABAQUS sheet of elasticity. In order to develop the plastic range in ABAQUS, the yield strength and the plastic strain corresponds to each increment of stress which are needed to define in its ABAQUS sheet. All mechanical properties have been used of this model from experiment test.

4-2-3. Mesh of FE model

The mesh of the finite element model is an important element, because a poor mesh could show us unrealistic results. A mesh density study was performed to achieve a fine mesh of the model, as illustrated in figure (4-2).
4-2-4. Boundary condition and loading

For the finite element to simulate this experiment, the two ends of FE model are fixed from displacing or rotating them in the three directions X, Y and Z. The boundary conditions applied in this simulation are illustrated in figure (4-3). The inner pressure was applied to the inner surface of the finite element model of penstock.

![Figure (4-3), Boundary condition and applied inner pressure.](image)

4-2-5. Initial residual stresses for first load

To simulate the effect of residual stresses on the behavior of weld joints of the first load, 40% of yield strength was added to each weld joint as a predefined field, and the six values of von Misses stresses were defined in its ABAQUS sheet.
As it will be shown later on (Chapter 5), the resulted residual stress after first load-unload is much lower than the initial residual stresses, and to simulate the effect of initial residual stresses on weld joints for the second load, we assume the value of initial residual stresses for the second load is equal or a little bit higher than that one used for the first load.
4-3. Experimental work of hydrostatic test of a full-scale model of penstock

4-3-1. Introduction

The application of High Strength Low Alloyed (HSLA) steels for production of pressure vessels is achieved by development of weld consumables (electrodes, wires and flax) with low strength and higher plasticity compared with Base Metal (BM) [44]. The combination of such weld constituents tensile properties is known as the under matching (UM) effect. Plastic deformations during the testing and exploitation procedures will be concentrated in Weld Metal (WM) leaving the BM with elastic deformations. The applied stress level, which will produce plastic deformations, is lower than the one declared with yield strength of BM. Welded joints are places with non-uniform stress distribution because of stress concentration and residual stresses introduced by the welding procedure. Simultaneously introducing different influencing factors will produce difficulties by stress calculation of welded joints. The calculation and design of pressure vessels have to take into account all influencing factors on stress distribution in order to achieve good use of materials and needed safety. On the produced pressure vessels there are possible deformation measurements, using different techniques and needed for stress calculations and assessment of welded joints behavior under different service conditions [45].
Figure (4-6), Design of penstock segment full-scale model: 1-mantle; 2-lid; 3-stiffener; 4-supports, L-Longitudinal, C-Circular; MAW – shielded manual arc welding (M); SAW-submerged arc welding (S). [2]

All welded joints on the model were controlled visually, ultrasonically, with radiography satisfying IIS/IWW requirements.

The prototype was produced using weldable high strength low alloyed steel (HSLA) Sumiten 80P (SM 80P) produced in the Japanese steel factory Sumitomo. Steel SM 80P belongs to HT80 steel with tensile strength above 800 MPa and yield strength above 700MPa. Tensile properties were achieved by quenching and tempered technology which requires strong obeying of welding procedures. The MAW weldments were done using basic low hydrogen electrode LB118, the SAW were using core wire U8013 plus M38F flux, produced by “Cobe Steel”, Japan. Certified welders were used to weld the prototype and later, the penstock. Trial samples for additional investigation were welded parallel with a prototype and were tested after hydraulic testing of the model [46].

Typical chemical composition of SM 80P steel plates and its weld metals is presented in Table 1, and mechanical properties in Table 2.
Table (4-1), Chemical composition of SM 80P steel and of MAW and SAW weld metals

<table>
<thead>
<tr>
<th>Element</th>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Cu</th>
<th>Cr</th>
<th>Ni</th>
<th>Mo</th>
<th>V</th>
<th>B</th>
<th>Ceq</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM 80P</td>
<td>0.10</td>
<td>0.30</td>
<td>0.90</td>
<td>0.01</td>
<td>0.008</td>
<td>0.24</td>
<td>0.48</td>
<td>1.01</td>
<td>0.47</td>
<td>0.03</td>
<td>0.0016</td>
<td>0.5</td>
</tr>
<tr>
<td>Weld metal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAW</td>
<td>0.06</td>
<td>0.53</td>
<td>1.48</td>
<td>0.011</td>
<td>0.005</td>
<td>-</td>
<td>0.24</td>
<td>1.80</td>
<td>0.43</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>SAW</td>
<td>0.07</td>
<td>0.37</td>
<td>1.87</td>
<td>0.01</td>
<td>0.011</td>
<td>-</td>
<td>0.44</td>
<td>0.13</td>
<td>0.73</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table (4-2), Mechanical properties of SM 80P steel and of MAW and SAW weld metals

<table>
<thead>
<tr>
<th>Material</th>
<th>Direction</th>
<th>Tensile test</th>
<th>Charpy impact test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Y.S., MPa</td>
<td>U.T.S., MPa</td>
</tr>
<tr>
<td>SM 80P</td>
<td>rolling</td>
<td>755 - 794</td>
<td>804 - 834</td>
</tr>
<tr>
<td></td>
<td>cross rolling</td>
<td>755 - 794</td>
<td>795 - 834</td>
</tr>
<tr>
<td>Weld metal</td>
<td>MAW</td>
<td>722</td>
<td>810</td>
</tr>
<tr>
<td></td>
<td>SAW</td>
<td>687</td>
<td>804</td>
</tr>
</tbody>
</table>

4-3-2. Hydrostatic testing of the model was done in three stages as follows:

1. The checking of the measuring system, increasing the inner pressure from 0 to 30 Bars
2. Increasing the inner pressure from 0 to 92 Bars in order to produce the hoop stress into the mantel which corresponds to the service stress and unloading
3. Overloading of the model with 30% in correspondence with mantel service stress by increasing the inner pressure from 0 to 123 Bars.

Strain Gauges (SG) and Moiré grids measured the deformations of the model. On the outer side of the model 51 strain gauges with different characteristics were placed. Figure 4-7 presents the instrumentation on the developed model mantle with the scheme of cut samples for specimens planned for testing after the hydrostatic test [44].

59
Figure (4-7), Instrumentation and specimens sampling in penstock model static pressure test.

4-3-3. Stress and strain distribution in elastic range of penstock model test

The ratio between the mantel thickness ($t = 47$ mm) and inner diameter ($d = 4200$ mm) is 0.01 so the model can be treated as a thin shelled pressure vessel with similar strain values on the inner and outer surface. Strain measurement on the outer surface is much easier than on the inner where strain gauges have to be properly protected against the pressure.

On the outer surface on the model for linear elastic behavior are the following valid formulas for stress and strains:
a) Stress

Hoop stress \[ \sigma_t = p \frac{2d^2}{D^2 - d^2} \]  

Axial stress \[ \sigma_z = p \frac{d^2}{D^2 - d^2} \]  

Radial stress \[ \sigma_r = 0 \]

b) Strain

Hoop strain \[ \varepsilon_t = \frac{1}{E} \left( \sigma_t - \nu \sigma_z \right) \]  

Axial strain \[ \varepsilon_z = \frac{1}{E} \left( \sigma_z - \nu \sigma_t \right) \]

Radial strain \[ \varepsilon_r = \frac{1}{E} \nu (\sigma_t + \sigma_z) \]

Where \( D = 4294 \text{ mm} \) – outer diameter, \( d = 4200 \text{ mm} \) – inner diameter, \( E = 210 \text{ GPa} \) – modulus of elasticity, \( \nu = 0.3 \) - Poison ratio.

By replacement of upper values stress and strain values are dependent on inner pressure given as:

\[ \sigma_t = 44.186 p, \sigma_z = 22.093 p, \sigma_r = 0, \varepsilon_t = 178,85 \cdot 10^{-12} p \]

All formulas are valid for a model produced of isotropic material without taking into account the residual stresses, stress concentration, presence of welded joints, and deviation of ideal geometric shape.

According to Misses hypothesis ideal stress \( \sigma_i \) can be calculated as

\[ \sigma_i = \sqrt{\frac{1}{2} \left[ (\sigma_t - \sigma_r)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_t)^2 \right]} \]

By replacing for \( \sigma_r = 0 \) final expression for ideal stress will be as
\[ \sigma_i = \sqrt{\sigma_i^2 - \sigma_i \sigma_z + \sigma_z^2} = 38.266 \text{ p} \] (4-5)

Ideal strain can be calculated according to the well known expression as

\[ \varepsilon_i = \frac{\sqrt{2}}{2(1 + \nu)} \sqrt{\left(\varepsilon_t - \varepsilon_z\right)^2 + \left(\varepsilon_z - \varepsilon_r\right)^2 + \left(\varepsilon_r - \varepsilon_t\right)^2} = 182.22 \text{ p} \] (4-6)

Taking inner pressure \( p \) as a parameter into the expression (4-3) one can obtain the following relationships between stress and strain I hoop and axial direction

\[ \sigma_t = 247.05 \cdot 10^9 \varepsilon_t; \sigma_z = 525.02 \cdot 10^9 \varepsilon_z; \sigma_i = 210 \cdot 10^9 \varepsilon_i \] (4-7)

By solving expressions (4-2) it is possible to calculate stresses in hoop and axial direction from measured strain gages strains \( \varepsilon_t \) and \( \varepsilon_z \) as

\[ \sigma_t = \frac{E}{1 - \nu^2} \left(\varepsilon_t + \nu \varepsilon_z\right) \]

\[ \sigma_z = \frac{E}{1 - \nu^2} \left(\varepsilon_z + \nu \varepsilon_t\right) \] (4-8)

Which are valid for plain stress condition (\( \sigma_r = 0 \))

Using expressions (4-8) it is possible to calculate the stresses in hoop and axial direction for measured strains \( \varepsilon_t \) and \( \varepsilon_z \). Ideal strain \( \varepsilon_i \) can be calculate after simplifying the expression (4-6) as

\[ \varepsilon_i = \frac{E}{1 - \nu^2} \sqrt{\left(1 - \nu + \nu^2\right) \cdot \left(\varepsilon_t^2 + \varepsilon_z^2\right) - (1 - 4\nu + \nu^2) \varepsilon_t \varepsilon_z} \] (4-9)

Using above equations it is possible to have the following stress – strain distributions:

\( \sigma_t - \varepsilon_t; \sigma_z - \varepsilon_z \) and \( \sigma_i - \varepsilon_i \).
During the hydrostatic model test there are on some places total strains for which the stresses will be out of linear elastic stress – strain distribution. For such cases we need the relationships taking into account the plastic deformation and strain hardening effect where the ideal stress can be expressed using the Romberg – Osgood as

$$\sigma_i = A\varepsilon_{ipl}^n$$  

(4-10)

Where $A$ and $n$ are the strength and strain hardening coefficients obtained from uniaxial tensile tests of the tensile specimens representing the observed place usually the weld metal\[47\]. Ramberg - Osgood coefficients for all welded joints on the penstock model are presented in Table (4-3).

<table>
<thead>
<tr>
<th>Weld joint on the Penstock model</th>
<th>Weld designation</th>
<th>Strength coefficient $A$, MPa</th>
<th>Strain hardening $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal SAW</td>
<td>L-SAW</td>
<td>1217.2</td>
<td>0.076</td>
</tr>
<tr>
<td>Longitudinal MAW</td>
<td>L-MAW</td>
<td>1041.8</td>
<td>0.044</td>
</tr>
<tr>
<td>Circular SAW</td>
<td>C-SAW</td>
<td>1232</td>
<td>0.079</td>
</tr>
<tr>
<td>Circular MAW</td>
<td>C-MAW</td>
<td>1029.8</td>
<td>0.047</td>
</tr>
</tbody>
</table>

In order to use the equation (4-10), one needs to express the total ideal strain $\varepsilon_i$ as the sum of elastic and plastic part as

$$\varepsilon_{i\text{-total}} = \varepsilon_{i\text{elastic}} + \varepsilon_{i\text{plastic}}$$  

(4-11)
In order to simplify the procedure for cylindrical pressure vessel it is valid ratio

\[ m = \frac{\sigma_z}{\sigma_t} = 0.5 \]  \hspace{1cm} (4-12)

Expression (4-5) can be written in simple form using the parameter \( m \) as

\[ \sigma_i = \sigma_t \sqrt{1 - m + m^2} \]  \hspace{1cm} (4-13)

For plastic deformation Poisson’s coefficient (\( \nu = 0.5 \)), ideal plastic deformation can be expressed as [48]

\[ \varepsilon_{iplastic} \equiv \frac{\sigma_i}{E} = \frac{\sigma_t}{E} \sqrt{1 - m + m^2} \]  \hspace{1cm} (4-14)

And hoop plastic strain is

\[ \varepsilon_{t\text{plastic}} \equiv \frac{1}{E} (\sigma_t - 0.5\sigma_z) = \frac{\sigma_t}{2E} (2 - m) \]  \hspace{1cm} (4-15)

Combining the expressions (4-12) and (4-13) it is possible to correlate the ideal plastic hoop strain and hoop plastic strain as

\[ \varepsilon_{iplastic} = \frac{2}{2-m} \sqrt{1-m+m^2} \cdot \varepsilon_{t\text{plastic}} \]  \hspace{1cm} (4-16)

Hoop plastic strain is

\[ \varepsilon_{t\text{plastic}} = \varepsilon_{t\text{total}} - \varepsilon_{t\text{elastic}} \]  \hspace{1cm} (4-17)

At the end the equation (4-10) can be expressed as

\[ \sigma_t = A \left( \frac{2}{2-m} \right)^n \left( 1 - m + m^2 \right)^{\frac{n-1}{2}} \varepsilon_{iplastic}^n \]  \hspace{1cm} (4-18)
4-4. Residual strength prediction of penstock

4-4.1 introduction

Safe life prediction of structural components is the main consideration for designing; pressure vessel should be able to sustain the design load (inner pressure) during its life time, where most structural components contain defects (flaws).

A serious structural problem that arises in the pressure vessel is the delayed time failure of pressure vessel due to sustained pressurization, even with inert environments. Failure can occur after only a few operational pressure cycles. In some cases through-thickness cracks have formed, and the vessel leaked under pressure. In other cases, small surface cracks grow to a critical size prior to becoming through-thickness flaw, resulting in a catastrophic failure. The significant parameters affecting the critical flaw size are applying stress level, the properties of material (fracture toughness), and the wall thickness of pressure vessel and the location of flaw. In order to predict the growth of flaw in pressure vessel using fracture mechanics, elastic stress intensity factors for brittle material, or using J integral and crack opening displacement of ductile materials.

4-4-2. Evaluation of critical crack size of surface flaw (point of instability)

In order to predict the residual strength of surface flaw by using a resistance curve of material and crack driving force curve of structure, the following proposed procedures are used:

Consider surface flaw geometry as shown in figure (4-8), where: (d) the depth of crack at the center, (2a) the length of surface.

![Figure (4-8), surface flaw geometry](image-url)
1-construct the $\sqrt[3]{J_R}$ curve of the material of the structure using suitable specimen
2-construct $\sqrt[3]{J_R}$ curves for the structure at various crack depths and applied stress using a suitable model
3-Determine the point of instability, which defined at the point of tangency between the crack driving force curves and J-R curve.

4-4-3. Experimental procedure:

The approach to welded structure designs is that the weld metal strength under matches the strength of the base metal. This means that the yielding will start in weld joint, and the base metal will start to yield when the strength of weld joint reaches (strain hardening) a level of base metal yield strength.

In these experiments the Sumiten 80P (SM 80P) grade steel plate (16 mm thick) was butt welded (X-shaped preparation) by submerged arc welding using consumables of 80B wire and MF38 flux, where these combinations under-matched weld joints are obtained.

Three tensile panels with surface cracks positioned in the base metal (BM), weld metal (WM), and heat-affected zone (HAZ) were tested at room temperature.

A semi-elliptical small and large surface crack (SSF, $d = 2.5:3$ mm, LSF, $d = 4.5:5$ mm) was produced by electrical discharge machine at BM, WM, and HAZ

The objective of this was to induce stable crack extension, and this test requires continuous measurement of force versus crack mouth opening displacement and crack extension was monitored during the test by the compliance method.

Figure (4-6), preparation samples for tensile panel test
4-4-4. Mechanical properties and chemical composition of materials:

Under matched weldments are recommended for high strength low alloy steel (HSLA) with yield strength of above 700 MPa in order to avoid cold cracks. The occurrence of defects cannot be completely avoided [49].

Table (4-4), Mechanical properties of materials

<table>
<thead>
<tr>
<th>Mechanical properties (MPa)</th>
<th>Base metal (BM)</th>
<th>Weld metal (WM)</th>
<th>Heat affected zone (HAZ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield strength (σ_y)</td>
<td>750</td>
<td>718</td>
<td>734</td>
</tr>
<tr>
<td>Tensile strength (σ_ut)</td>
<td>820</td>
<td>791</td>
<td>800</td>
</tr>
</tbody>
</table>

Table (4-5), chemical composition of materials

<table>
<thead>
<tr>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Cr</th>
<th>Ni</th>
<th>Mo</th>
<th>V</th>
<th>Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.20</td>
<td>0.23</td>
<td>0.009</td>
<td>0.018</td>
<td>1.24</td>
<td>3.1</td>
<td>0.29</td>
<td>0.05</td>
<td>0.08</td>
</tr>
</tbody>
</table>

4-4-5. Construct of J-R curve (resistance curve of material) of BM, HAZ and WM

According to ASTM standard E-1820[50] using the following equations to calculate J integral of (BM, HAZ, WM):

\[ J = J_{el} + f_{pl} (4-19) \]

Where:

\[ J_{el} \] = elastic component of J integral
\[ J_{pl} = \text{plastic component of J integral} \]

\[ J_{el} = \frac{K^2 (1 - v^2)}{E} \]  \hspace{1cm} (4-20)

Where \( K \) is the stress intensity factor and calculated as follows:

\[ K = \frac{P}{B \sqrt{Ww}} f'(a/w) \]  \hspace{1cm} (4-21)

Where:

- \( P \) = applied load (KN)
- \( f'(a/w) \) = dimensionless function of \( a/w \)
- \( W \) = width of the specimen (mm)
- \( B \) = thickness of specimen (mm)

\[ J_{pl} = \frac{\eta A_{pl}}{B b_0} \]  \hspace{1cm} (4-22)

Where:

- \( \eta = 3.785 - 3.101 \left( \frac{a}{w} \right) + 2.018 \left( \frac{a}{w} \right)^2 \)
- \( A_{pl} = \text{plastic area under load-CMOD curve} \)
- \( b_0 = W - a_n \)

4-4-6. Construct crack driving force curve of BM, HAZ and WM

Crack driving forces are calculated for various values of crack depth ratio (\( d / h \)) for pressure vessels with the shell parameter equal to zero (\( \lambda = 0 \))

Crack driving force of cylindrical shell could be calculated using the model proposed by Ratwani, Erdogan and Irwin [51].
\[ J^* = \frac{JE}{4a\sigma^2} \]  
(4-23)

And these crack driving force curves were determined for various values of stresses ratio (PR / ho_y).

**Figure (4-7) CDF of base metal of penstock**

4-4-7. Determine the point of instability

In order to determine the point of instability of a structure, the resistance curves (J-R curve) are superimposed on the diagram of crack driving force curve to find the point of instability (the point of tangency between J-R curve and CDF.)
5-Results and discussion

5-1.Finite element analysis (ABAQUS) of full-scale model of penstock

The FEA results include:

1- Von Misses stresses distribution of full-scale model of FEA model, which is the model loading by inner pressure in two steps, (first load-unload, second load-unload).
2- Von Misses stress-strain curves (FL-UNL, SL-UNL).
3- Von misses stress-Inner pressure curves (FL-UNL, SL-UNL).
4- Inner Pressure-von Misses strain curves (FL-UNL, SL-UNL).
5- Hoop stresses-strain curves (FL-UNL, SL-UNL).

5-1-1. Von Misses stresses distribution of FE model for FL (without RS).

Figure (5-1) showed the von Misses distribution of the finite element model for first load as calculated in ABAQUS software. The highest stresses was in the weld joint (LSI SAW), and the base metal at that side. This concentration of stress is due to the geometrical shape of the model, which exerts more stresses (compression) on that side, and the geometry of the model tends to be ideal.
5-1-2. Plastic strain (FL-UNL).

As indicated in figure (5-2), the plastic strain is only initiated in the weld joint (LS1 SAW). This behavior is due to the lower yield strength of the joint and its location in the stress concentration region.

Figure (5-2), plastic deformation of FE model (FL-UNL, P=14.5MPa).


As the internal pressure increased in the second load of FE model, the level of von Misses stress will be increased, and the distribution of stress has not been changed compared to the first load except the behavior of weld joint (LS1 SAW), which has lower stress than the base metal at that side of stress concentration region due to the effect of initiation of plasticity as indicated in figure (5-3).
Figure (5-3), von Misses stresses distribution of FE model of second load, (P=18.5MPa)

5-1-4. Plastic strain (SL-UNL).

As illustrated in figure (5-4), the levels of von Misses stresses have exceeded the yielding of the base metal and weld joints at that side of the stress concentration region and the plasticity initiated and spreads in base metal and weld joints in this area.

Figure (5-4), plastic deformation of FE model (SL-UNL, P=18.5MPa).
5-1-5. Von Misses stress-strain curve of weld joint LSI SAW without RS (FL-UNL, SL-UNL).

Figure (5-5) illustrates the behavior of von Misses stress-strain curve of the weld joint (LS1SAW) for first load-unload and second load-unload as calculated in ABAQUS software. This behavior showed the linearity of the stress-strain curve of the loading and unloading behavior for the first and second load.

![Figure (5-5), Von Misses stress-strain behavior LS1 SAW without RS as calculated in ABAQUS.](image-url)
5-1-6. Von Misses stress-inner Pressure curve of weld joint LSI SAW without RS, (FL-UNL, SL-UNL).

Figure (5-6), showed the behavior of von Misses stresses with loading and unloading of the FE model by inner pressure. As the inner pressure increases the von Misses stresses increases to the yield point of the weld joint, then the changing of Von Misses stresses will be lower. For unloading, the behavior will be linear, until the effect of residual stresses and then it will be non-linear.

Figure (5-6), Von Misses-Inner Pressure behavior of WM LSI SAW as calculated in ABAQUS.
5-1-7. Inner Pressure-Von Misses strain curve of weld joint LS1 SAW without RS (FL-UNL, SL-UNL).

The behavior of the von Misses strain with inner pressure as calculated in ABAQUS is illustrated in figure (5-7). This behavior showed linearity during loading and unloading with a little bit of change during plasticity.

![Graph showing inner Pressure-Von Misses strain](image)

Figure (5-7), inner Pressure-Von Misses strain of LS1 SAW as calculated in ABAQUS.

The behavior of hoop stress-strain curve as indicated in figure (5-8). The yielding for the first load starts at 13.34 MPa of inner pressure (531.515 MPa of hoop stresses), while for the second load the plastic deformation initiated at 14.8 MPa of inner pressure (586.149 MPa of hoop stresses).

Figure (5-8), Hoop stresses-strain curve of WM LS1 SAW as calculated in ABAQUS.
5-1-9. Von Misses stresses distribution of FE model for FL (with RS).

Figure (5-9) showed von Misses stresses distribution of finite element model as calculated in ABAQUS, the highest stress has been in weld joints at the stress concentration side. This high level of stress is due to the effect of initial residual stresses (40% of yield strength) and the geometric shape of the model.

Figure (5-9), von Misses stresses distribution of FE model for first load with RS (P=11.2MPa).
5-1-10. Plastic strain (FL-UNL, with RS).

Figure (5-10) showed the initiation of plasticity after first load of FE model, where the plasticity initiated is in the weld joint LS1 SAW. This behavior is due to the lower yield point and its location of this joint.

Figure (5-10), plastic strain of WM LS1 SAW after FL (P=11.2MPa).
5-1-11. Von Misses stresses distribution of FE model for SL (with RS).

Figure (5-11) showed von Misses distribution of FE model for second load with residual stresses, as the inner pressure increased for the second load, the highest von Misses stresses were still in the weld joints at the concentration stress side with a considerable increasing of von Misses stresses in the base metal on that side.

Figure (5-11), Von Misses distribution of FE model for SL with RS (P=14.4MPa).
5-1-12. Plastic strain (SL-UNL, with RS).

As the inner pressure increased for second load, the plastic strain initiated in the other weld joints was at the shorter side CMAW, LS3 SEW.

Figure (5-12), plastic strain of WM LS1 SAW after SL (P=14.4MPa).
The behavior of the von Mises stress-strain curve of weld joint LS1 SAW with residual stresses is similar to the behavior without residual stresses, but it yields at a lower level of inner pressure due to the effect of residual stresses as illustrated in figure (5-13).

Figure (5-13), von Mises stresses-strain curve of WM LS1 SAW with RS as calculated in ABAQUS.

The behavior of the hoop-stress-strain curve of the weld joint LS1 SAW with residual stresses showed that the plastic strain for first load was in the direction of axial stresses and not in a circumferential direction (there is no plasticity that appears for first load in hoop stress-strain curve) as indicated in figure (5-14). This behavior is due to the geometric shape of the model (5º angle), which exerts more compression in the axial direction.

![Hoop stress-strain curve](image_url)

Figure (5-14), Hoop stresses-strain of WM LS1 SAW with RS as calculated in ABAQUS.
5-2. Experimental results with numerical calculations of full-scale model of penstock.

There are three different stress-strain distribution treatments. The first one is the treatment of model as ideal cylindrical pressure vessel which behaves according to the linear elastic formulas (4-1), (4-2) and (4-3). The second distribution will be obtained from strain gauge measurements in both loading – unloading regimes using elastic – plastic formulas. The third treatment will be the use of finite element ABAQUS calculations.

The first treatment used for the ideal cylinder pressure vessel can be presented in Table (5-1). Table (5-1), Stress – strain distribution for an ideal cylinder without taking into account residual stresses and stress concentration

<table>
<thead>
<tr>
<th>Pressure MPa</th>
<th>Strain, $\mu$m/m</th>
<th>Stresses, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_1$</td>
<td>$\varepsilon_2$</td>
</tr>
<tr>
<td>0,00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0,50</td>
<td>89</td>
<td>21</td>
</tr>
<tr>
<td>2,95</td>
<td>528</td>
<td>124</td>
</tr>
<tr>
<td>4,40</td>
<td>787</td>
<td>185</td>
</tr>
<tr>
<td>5,90</td>
<td>1055</td>
<td>248</td>
</tr>
<tr>
<td>7,35</td>
<td>1315</td>
<td>309</td>
</tr>
<tr>
<td>8,35</td>
<td>1493</td>
<td>351</td>
</tr>
<tr>
<td>9,05</td>
<td>1619</td>
<td>381</td>
</tr>
<tr>
<td>9,80</td>
<td>1753</td>
<td>412</td>
</tr>
<tr>
<td>10,80</td>
<td>1932</td>
<td>454</td>
</tr>
<tr>
<td>11,50</td>
<td>2057</td>
<td>484</td>
</tr>
<tr>
<td>12,05</td>
<td>2155</td>
<td>507</td>
</tr>
</tbody>
</table>
Table (5-2). Stress – Strain distribution obtained from Strain Gages readings

<table>
<thead>
<tr>
<th>Pressure MPa</th>
<th>Strain, µm/m</th>
<th>Stress, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_t$</td>
<td>$\varepsilon_z$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.50</td>
<td>89</td>
<td>21</td>
</tr>
<tr>
<td>2.95</td>
<td>593</td>
<td>124</td>
</tr>
<tr>
<td>4.40</td>
<td>855</td>
<td>185</td>
</tr>
<tr>
<td>5.90</td>
<td>1202</td>
<td>248</td>
</tr>
<tr>
<td>7.35</td>
<td>1616</td>
<td>309</td>
</tr>
<tr>
<td>8.35</td>
<td>1846</td>
<td>351</td>
</tr>
<tr>
<td>9.05</td>
<td>1994</td>
<td>379</td>
</tr>
<tr>
<td>9.50</td>
<td>277</td>
<td>21</td>
</tr>
<tr>
<td>0.00</td>
<td>183</td>
<td>0</td>
</tr>
<tr>
<td>0.50</td>
<td>277</td>
<td>21</td>
</tr>
<tr>
<td>2.95</td>
<td>578</td>
<td>124</td>
</tr>
<tr>
<td>7.35</td>
<td>1534</td>
<td>309</td>
</tr>
<tr>
<td>9.05</td>
<td>1879</td>
<td>379</td>
</tr>
<tr>
<td>9.80</td>
<td>2060</td>
<td>412</td>
</tr>
<tr>
<td>10.80</td>
<td>2301</td>
<td>454</td>
</tr>
<tr>
<td>11.50</td>
<td>2484</td>
<td>484</td>
</tr>
<tr>
<td>12.05</td>
<td>2649</td>
<td>507</td>
</tr>
<tr>
<td>12.05</td>
<td>2654</td>
<td>507</td>
</tr>
<tr>
<td>10.80</td>
<td>2413</td>
<td>454</td>
</tr>
<tr>
<td>9.00</td>
<td>2064</td>
<td>379</td>
</tr>
<tr>
<td>7.35</td>
<td>1717</td>
<td>309</td>
</tr>
<tr>
<td>2.95</td>
<td>842</td>
<td>124</td>
</tr>
<tr>
<td>0.50</td>
<td>362</td>
<td>21</td>
</tr>
<tr>
<td>0.00</td>
<td>273</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure (5-15). Relationships Stress – Strain for ideal cylinder

Figure (5-16), BM Von Misses Stress vs. Inner Pressure comparison with FE calculations

Figure (5-16) presents the Von Misses stress dependence with applied inner pressure for an ideal cylinder without taking into account the residual stress and stress concentration, together with the calculations obtained using the strain gauges readings in two loading-unloading sequences and finally with finite elements (FE) calculations. FE calculations are showing the linear relationship until they reach the yield strength of the base metal and...
further plastic behavior and linear unloading. The pressure at which yield strength is reached is about 15 MPa, the pressure which was not used by hydraulic penstock model testing.

Figure (3-17) shows good agreement of Von Misses stress-strain dependence obtained for different calculation methods.

Figure (5-18). Relationships between inner pressure and von Misses strain for Penstock Model.
Figure (5-18) presents the relationships of inner pressure depending on different ways of presenting the von Misses strain. Von Misses strain is calculated for an ideal cylinder, according to the strain gauge readings and finally using finite element calculations. Both relationships obtained for an ideal cylinder and FE calculations show linear behavior for inner pressure of 12.05 MPa. Strain gauge readings show little deviation from linearity due to the stress concentration, and geometric imperfections.

![Figure (5-18), Stress – Strain distribution for penstock model](image)

Figure (5-18) shows linear relationships between the von Misses stress and strain obtained for an ideal cylinder, for strain gauge readings and for FE calculations. It looks like the ideal cylinder and strain gauge readings highly agree but FE calculations show slightly different behavior.
Figure (5-20) shows similar behavior of the linear relationship for ideal cylinder and FE calculations and a slight deviation for results obtained from strain gauges readings. After unloading from 9.05 MPa there was small residual stress which was the starting point for the next loading to 12.05 MPa. Again after unloading was calculated, there was a similar amount of residual stress. The reason for this residual stress is geometrical deviation from the ideal model shape.
Figure (5-21). Von Misses Stress – Strain relationships for SMAW weld joint LS1

Figure (5-21) shows the relationships von Misses stress – strain for weld joint LS1 obtained using strain gauge readings, finite element calculations without taking into account residual stresses and geometrical imperfections and finite element calculations using residual stresses into account. The position of weld joint LS1 is on the shorter side of the upper cylinder with a slope of 5° and it is logical to expect high tensile stress and strains because under the inner pressure, the cylinder is trying to reach the ideal cylinder. Good agreement between SG readings and FE calculations is obtained using much higher inner pressure than the one used by experimental testing where the model was tested to 12.05 MPa. Using residual stress of 273 MPa it will once again be in good agreement. Residual stress of 273 MPa is obtained as 40% of yield strength of SMAW weld joint.
Figure (5-22). Von Misses Stress – Strain relationships for SMAW weld joint LS2

Figure (5-22) shows Von Misses stress strain relationships obtained from SG readings, without taking into account residual stresses and with taking residual stress of 273 MPa. Also it is showing the inner pressure correlating with the von Misses strain. The weld joint LS2 is on the longer side of the upper cylinder and under the inner pressure will show lower stress because the cylinder is trying to reach the ideal shape.
Because of the positioning of the two SMAW weld joints LS1 and LS2, it is easy to see the linear relationship of Stress – strain for the weld joint on the longer side of the upper cylinder of the penstock model and the plastic behavior of LS1 which is positioned on the lower side. Weld joint LS2 has not reached the yield strength of the SMAW welded joint LS2.

After analyzing the upper cylinder of the penstock model, which is with a 5° deviation from the ideal cylinder geometry causing the additional stress concentration compared with the middle and lower cylinder of the model. Let us start with the SMAW joint L3 positioning on the similar side like SMAW L1.
Figure (5-24). Hoop Strain against the applied inner pressure during both loading sequences.

Figure (5-24) shows the hoop strain response on the applied inner pressure in first and second loading sequence. According to the hoop strain readings one can conclude that Von Mises stress will reach the yield stress of the welded joint.

Figure (5-25) Hoop Stress – Strain distribution for L3 SMAW joint during loading in two sequences.
Figure (5-25) presents the Hoop stress – strain distribution for L3 SMAW joint together with the yield strength of weld metal. Hoop stresses after the inner pressure reached 9.8 MPa are higher than the yield stress of the SMAW weld metal.

L3 SMAW

![Graph showing Hoop stress and strain distribution](image)

Figure (5-26). Von Misses Stress – Strain distribution for L3 SMAW joint compared with FE calculations

On the figure (5-26), are presented Von Misses stress - strain distributions obtained from the SG readings together with the FE calculations. FE is taking into account initial residual stress of 293 MPa reaching the yield strength for inner pressure of 12.05 MPa. It can be concluded that there is good agreement reached between the calculations using SG readings and FE calculations using initial residual stress.
Figure (5-27), Inner pressure – Hoop strain for L4 MAW

Figure (5-28), Hoop Stress – Hoop strain for L4 MAW
**Figure (5-29), Von Misses Stress – Strain for L4 MAW**

**Figure (5-30), Comparison between the welded joints stress–strain distribution in the second cylinder.**
Now let us move on to the circular welded joints. For circular CM MAW weld joint is presented in the next diagrams as follow:

**Figure (5-31).** Inner pressure vs. Hoop strains for CM MAW circular weld joint

**Figure (5-32).** Hoop Stress vs. Hoop strains for CM MAW circular weld
Figure (5-33), Von Misses Stress vs. Hoop strains for CM MAW circular weld together with FE calculations.

Figure (5-34), Comparison of Von Misses Stress – Strain relationships for two circular weld joints.

97
5-3. Residual strength prediction of penstock by using R-curve:

5-3-1. Stresses-CMOD (crack mouth opening displacement), relationship for small and large surface flaw of the base metal, weld metal and heat affected zone.

Figure (5-35), stresses-CMOD relationship of SSF.

Figure (5-36), stresses-CMOD relationship of LSF.
5-3-2. J-R curves of small and large surface flaw of BM, WM and HAZ.

When the crack driving force equals or exceeds the fracture toughness of the material, the crack starts to grow, therefore the J-R curves for different components (BM, WM, HAZ) and different depth (SSF, LSF) were determined according to the previous method and got the following results, as indicated in figure (5-37).

The results of the procedure of J-R curves for large surface flaws as indicated in figure (5-37), the base metal showed higher resistance to propagation of the crack and heat affected zone which had lower resistance. The heterogeneity of microstructure in HAZ plays a main role in this.

As indicated in figure (5-38) the J-R for small surface flaw curve of weld metal showing lower resistance to the crack growth while the base metal with SSF shows higher resistance to the crack growth. This means the existing small surface flaw in the weld metal will grow faster than others and the leakage and failure is expected to happen in this location.
Figure (5-39) indicated that, for the measured length of surface crack with large surface flaw (depth of crack, \( d = 4.741 \) mm), the point of instability was reached at pressure 11.95 MPa, and for a small surface flaw (depth of crack, \( d = 2.376 \) mm), the point of instability reached a pressure of 15.77 MPa, and the crack will be stable up to:

- SSF \( (d/h) = 0.245 \) and \( da = 1.544 \) mm
- LSF \( (d/h) = 0.38 \) and \( da = 1.339 \) mm

![Graph](image-url)

Figure (5-39), determining the point of instability of BM of penstock.
Figure (5-40) showed that, the point of instability of large surface flaw of weld metal reached a pressure of 11.16 MPa and crack growth will be stable up to a depth ratio of 0.46 and crack extension (da = 2.416mm), while for a small surface flaw, the pressure of instability was 13.01 MPa and the crack will be stable up to a depth ratio of 0.31 and crack extension (da = 2.854mm).

Figure (5-40), determination the point of instability of WM of penstock.
As indicated in figure (5-41) the point of instability of small surface flaw reached a pressure of 15.27 MPa and the crack will be stable up to a depth ratio of 0.23 and maximum stable crack extension was \( (d_a = 1.652 \text{mm}) \). For a large surface flaw the pressure of instability was 10.23 MPa, and crack will be stable up to a depth ratio of 0.42 and crack extension \( (d_a = 1.56 \text{mm}) \).
Conclusion

Finite element analysis (ABAQUS software) for a full-scale model of penstock has been performed in order to simulate a hydrostatic test of the experimental model of the pressure vessel. Von Misses stress-strain distributions as calculated in ABAQUS, has been compared with stress-strain distribution of the ideal cylindrical model of a pressure vessel, and distribution obtained from strain gauge measurements in both loading – unloading regimes. For residual strength prediction and structural integrity assessment of penstock, the experiment investigation of specimens carried out by the notched tensile panel test of HSLA with under matched weld metal (4.3%) , to study the effect of the surface crack in each part of weld joint (BM, HAZ, WM) on fracture properties. This type of steel is recommended to avoid cold cracking, but due to heterogeneity of the microstructure and of mechanical properties of the weld joint, defects cannot be avoided completely. Therefore, adequate crack resistance properties are required in addition to tensile strength properties of the structure. Based on this study, the following conclusions are reached:

• The von Misses stresses distribution of the finite element model of penstock showed that, the highest stress level has been on the shorter side of the model (at a 5° angle), which represents the stress concentration region. This behavior is affected by the geometrical shape, which exerted more compression on that side in an axial direction.

• The upper segment of penstock on the shorter side represents the critical part of the structure. The experiment model and the finite element model have been shown the critical point of the penstock at that part (weld metal joint LS1SAW), where the plasticity started earlier than the other joint weld metal which has the same properties (yield strength).

• Good agreement between the strain gauge reading and finite element calculation, but higher levels of internal pressure are used for the finite element model in order to reach a closer level of stresses of the experimental test, due to the effect of residual stresses and geometrical imperfection.

• Using initial residual stresses of 40% of yield strength for weld metal joint in the finite element model reduces the level of internal pressure of the finite element model to reach a closer level of stresses of the experimental test. On the other
hand, the stress distribution of finite element model with initial residual stresses was completely different compared to the finite element model without initial residual stresses. The highest stress levels have been in weld metal joints on the shorter sides of the model (LS1 SA, LS3 SAW, CM MAW).

- In general, the existence of cracks reduces the mechanical properties of welded joints. This reduction in mechanical properties is influenced by depth of surface cracks. Weld metal and heat affected zones showed large decrease in these properties compared to the base metal.

- From the point of fracture properties in the weld metal, the behavior of this property will not be affected as much because of the resistance of crack growth on small and large surface cracks, which had approximately the same property of resistance of crack growth. This property in the base metal and heat affected zones are different from the weld metal joints. For small surface flaws, the base metal and heat affected zone showed better property of resistance to crack growth compared to large surface flaws, but due to the heterogeneity of the microstructure and properties of heat affected zones, the large surface flaw has the worst fracture property.
References

1. Somnath Chattopadhyaya, “Pressure Vessel Design and Practice”, Taylor&Francis Group, LLC, 2004


21- H. S. Lamba, “the J Integral Applied to Cyclic Loading,” Engineering Fracture Mechanics, 1975


23- H. S. Lamba, “the J Integral Applied to Cyclic Loading,” Engineering Fracture Mechanics, 1975


27- G.P. Nikihkov, Introduction to the Finite Element Method, Lecture Notes, University of Aizu, Japan

28- Chris Greenough., (C.greenough@rl.ac.uk)

29- The Von Misses yield criterion, Wikipedia

30- Abaqus, 2007 ABAQUS Users’ Manuel, Hibbit, Karlson and Sorenson Inc., Pawtucket, RL


33- L. Geschwindner, R. Disque, R. Bjorhovde, Factor Design of Steel Structure,” Prentic Hall, 1994


44- Anwendung der höchsten schweissgeeigneten Feinkornbaustähle St E47 und St E70, Das Richtlinie 011, Deutscher Anschuss für Stahlbau, 1974.


